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THE

PHYSICAL REVIEW.

THE INTERFERENCES OF THE DIRECT SPECTRA OF TWO GRATINGS.

By Carl Barus.[1]

1. *Introduction. Method.*—The chief purpose of the present paper is the search for phenomena similar to those recently discussed,[2] but in which the two spectra brought to interference are not inverted relatively to each other. Incidentally the strong interferences may have a value on their own account. It has been shown that the totality of the phenomena with spectra reversed on a transverse or a longitudinal axis are quite complicated and a series of companion researches in which similar results are aimed at, in the absence of inversion, is thus very desirable.

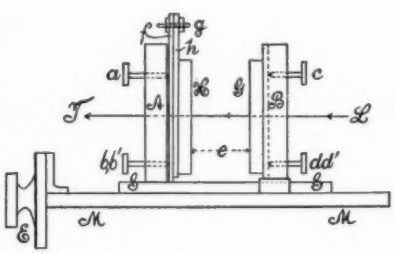

Fig. 1.

The apparatus (Fig. 1) is the same (nearly) as that shown in my last paper,[3] *MM* being the base of the Fraunhofer micrometer, *SS* the slide, *E* the micrometer screw. The brass capsules *A* and *B* are securely mounted on the slide *S*, free from the base *M*, and on the base *M* free from the slide *S*, respectively. Each capsule is provided with three adjustment screws relative to horizontal and vertical axes, *a*, *b*, *b'*, and *c*, *d*, *d'*, together with strong rearward acting springs, by which the gratings *G* and *H* at a distance *e* apart, may each be rotated slightly around a vertical or a horizontal axis (plane-dot-slot mechanism). The two gratings *G* and *H* must be identical, or very nearly so, as to the number of lines per inch and with their ruled faces toward each other.

[1] Abridged from a forthcoming Report to the Carnegie Institution of Washingotn, D. C.

[2] C. Barus, Am. Journ. Science, XL, pp. 486–498, 1915.

[3] C. Barus; Physical Rev., VII, pp. 79–86, 1916.

These faces, as well as the ruled lines, are to be in parallel. To secure the latter adjustment a bolt, g, normal to the face of the grating H, serves as an axis and an available tangent screw and spring (not shown) is at hand for fine adjustment. This device is of great importance in bringing the longitudinal axes of the two spectra due to G and H, into coincidence and a fine wire must be drawn across the slit of the collimator to serve as a guiding line through the spectrum. Any lack of parallelism in slit and rulings rotates the fringes.

The beam of light, L, either white or homogeneous as the experiment may require, is furnished by a collimator (not shown), which with the telescope at T (in plan, in Fig. 1*a*, at T or D) are the usual parts of a spectroscope. The collimator with slit is always necessary for adjustment. It may then be removed if the phenomenon is to be studied in the absence of the slit. The telescope is frequently replaced to advantage by a lens.

The adjustments in case of white light are simple and the interferences usually very pronounced, large and striking. Brilliant spectra, chanelled with vertical narrow black lines, are easily obtained when the longitudinal axes are placed accurately in coincidence by rotating the plate h carrying the grating H, on the plate f, around the axis g. If the gratings are quite identical the sodium lines will also be in coincidence. Otherwise the two doublets D_1D_2 and $D_1'D_2'$ of the two spectra (nearly identical in all their parts and in the same direction) are placed in coincidence by rotating either grating around a vertical axis. Thereupon the strong fringes will usually appear for all distances, e, less than 2 cm. These fringes are nearly equidistant, vertical and they intersect the whole spectrum transversely. They are not complicated with other fringes, as in the experiments of my last paper (l. c.). They increase in size till a single shadow fills the field of view, in proportion as the distance e is made smaller and smaller to the limit of complete contact. During manipulations great care must be taken to keep the angle of incidence, i, rigorously constant; *i. e.*, to avoid rotating both gratings together, or the apparatus as a whole, as this displaces the sodium doublets relative to each other and seriously modifies the equations.

2. *White Light. Colored fringes.*—The two sodium doublets seen in the arc spectrum are usually equally brilliant and but one set of strong fringes is present in the field of the telescope.

If both gratings are rotated, changing the angle of incidence from 0^0 to i^0, the fringes disappear from the principal focal plane, but reappear strongly in another focal plane (ocular forward or rearward). In such a case the D lines are no longer superposed. To be specific let i and i',

θ and θ', be the angles of incidence and diffraction at the two gratings in question, the angle between their ruled faces being $i - i'$. Let D and D' be the two grating constants, and nearly equal. Then for a given color, λ, in relation to the individual normals of the two gratings it may be shown that

$$\frac{i - i'}{D - D'} = \frac{\lambda}{D^2(1 - \cos\theta)}, \text{ nearly.} \qquad (1)$$

In case of the Wallace grating below $D = 1.75 \times 10^{-4}$, $\lambda = 58.93 \times 10^{-6}$, $i - i' = 10^4 \times 3.29\ (D - D')$. Thus if the enclosed angle $i - i'$ between the plates is one degree, or .0175 radian, $D - D' = 5.3 \times 10^{-7}$, about 3/10 per cent. of D, and equivalent to about 43 lines to the inch. With adequate facilities for measuring i this method may be useful for comparing gratings, not too different, in terms of a normal or standard, practically, since the finite equations may also be expanded. In a similar way the slight adjustments of the longitudinal axes of the two spectra may be made by rotating one grating around a horizontal axis, but this correction is less easily specified. Finally one should bear in mind that with film gratings there is liable to be an angle $i - i'$ between the adjusted plates.

The range of displacement of grating within which the fringes may be used with an ordinary small telescope, extends from contact of the two gratings, to a distance of $e = 2$ to 3 cm. beyond.

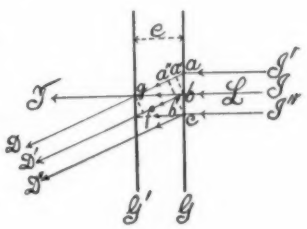

Fig. 1a.

In Fig. 1*a*, which is a plan of the essential planes of the apparatus, G, G' being the ruled faces of the gratings in parallel, I, I', I'', three impinging rays of white light diffracted into D, D', the points a, b, c, a', a'', b are in the same phase, so that the path difference of the rays from b at g and f is easily computed. If the single ray I is diffracted into D and D', or I and I' into D, I and I'' into D', etc., the micrometer value of a fringe for a color λ under normal incidence, is

$$\delta e = \frac{\lambda}{1 - \cos\theta}. \qquad (2)$$

For two colors λ and λ'

$$n\lambda = e(1 - \cos\theta) = eM;\ (n + n')\lambda' = e(1 - \cos\theta') = eM'$$

if n is the number of fringes between λ and λ'. Thus

$$n' = \frac{e\,M'\lambda - M\lambda'}{\lambda\lambda'} \qquad (3)$$

or the number of fringes increases as e is greater.

Equation (2) does not, as a rule, reproduce the phenomenon very well. Since the grating space D of the two gratings is rarely quite the same, the air plate enclosed, in case of apparent coincidence, of the sodium lines, is slightly wedge shaped, as in Fig. 2. Hence the two diffractions take place at incidences 0^0 and α^0, respectively, and the corresponding angles of diffraction will be θ and θ'. If we consider the two corresponding rays I and I'', diffracted at the first and second face, respectively, and coinciding at c in the latter, the points ab and a' (ba' normal to ac) are

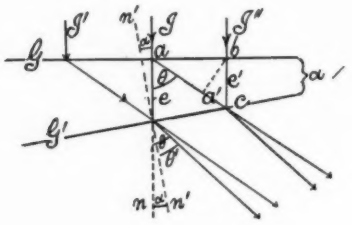

Fig. 2.

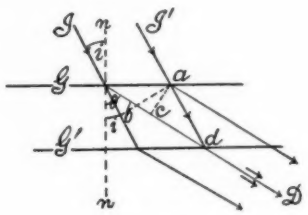

Fig. 2a.

in the same phase and we may compute the phase difference at the coincident points at c. The distance bc is

$$e' = e\,\frac{\cos\alpha\cos\theta}{\cos(\theta-\alpha)}$$

whence eventually

$$\delta e = \frac{\lambda\cos(\theta-\alpha)}{\cos\theta\cos\alpha\cdot(1-\cos\theta)}, \tag{4}$$

which changes into equation (3) when $\alpha = 0$ and $n = 1$. Fortunately this correction is, as a rule, small. In case of the Wallace gratings ($D = 1.75 \times 10^{-4}$ cm. for instance), if $\lambda = 58.93 \times 10^{-6}$, $\theta = 19^0\ 40^1$ or $\delta e = 1.01 \times 10^{-5}$; whereas if $\alpha = 5^0$, $\delta e = 1.04 \times 10^{-3}$; etc.

If the incidence is at an angle i and the plates are parallel, Fig. 2a, the inquiry leads in the same way to an equation of more serious import. If the gratings G and G' are at a distance e apart and the incident rays are I and I', the points a, b, c, are in the same phase. Hence the two rays leaving d and diffracted along D correspond to

$$\delta e = \frac{\lambda\cos i}{1-\cos(\theta-i)}. \tag{5}$$

Fig. 2c shows the variation of fringes with the angle of incidence i (equation (5)).

Hence if the angle of incidence is changed from -5^0 to $+5^0$, δe increases to nearly 3 times its first value. This therefore accounts for

the large discrepancies of δe found in the successive data below. To secure increased sensitiveness and to make the apparatus less sensitive to slight changes of i, this angle should be about 25^0, in which case δe is about three wave-lengths per fringe. But normal incidence is frequently more convenient.

Finally in Fig. 2*b*, if the angle of incidence is i and the two faces G and G' make an angle α with each other and are initially at a distance

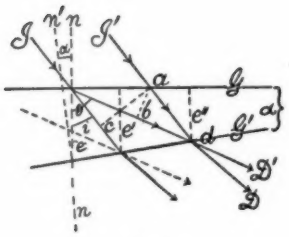

Fig. 2*b*.

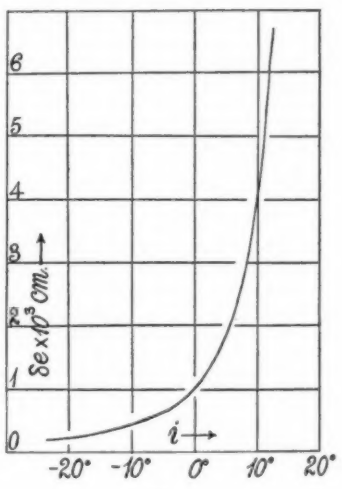

Fig. 2*c*.

e apart, changing successively to e' and e'', the points a, b, c, being in the same phase, the two rays D and D' leaving at d, at an angle $\theta - i$, will conform to

$$\delta e = \frac{\lambda \cos i \cos (\theta - \alpha)}{\cos \alpha \cos \theta (1 - \cos (\theta - i))}. \qquad (6)$$

This equation reproduces the preceding equation (5) if $\alpha = 0$ and the original equation (2) if $\alpha = i = 0$. It shows that a discrepancy or angle between the plates is of minor importance. Hence the change of this angle may be used to bring the sodium lines in coincidence when the gratings differ slightly in their grating constants D. On the other hand, changes of incidence i are of extreme importance.

Experiments made with the film gratings showed that equation (2) not only fits very badly, but that δe per fringe appears as a fluctuating quantity. Fringes were distinctly seen however within 3 cm. of displacement, by an ordinary telescope. I omit the data.

The reason for the lack of accord is given in equation (5) and (6) and Fig. 2*c*. To show that the irregularity of the above results is to be

sought in the accidental variations of the angle of incidence i at both gratings, the rough experiments in Table I. suffice.

TABLE I.

Effect of Changing i.

Ives's grating,[1] i negative, ocular inward,[2] $i = \pm 0°$; i positive, ocular outward. $10^3\,\delta e$ = .77, .78, .74; 1.18, 1.12; 1.69, 1.66 cm.

Thus, as equation (6) implies, small variations of i produce relatively large variations of δe and if i passes continuously through zero, from negative to positive incidence, δe increases continually and may easily be more than doubled. If the phenomenon is in focal planes in front of the principal plane (ocular in), δe is small and vice versa. Moreover this enormous discrepancy is quite as marked for thin glass (2 millimeters) as for thick glass plates (8 millimeters). Again the rather stiff screw of the micrometer, which twisted the whole apparatus slightly, was sufficient to introduce irregularity.

An additional result was obtained by placing a plate of glass between the two gratings G and G'. The effect was an unexpected *enlargement* of fringes, increasing with the thickness of the glass plate (.6 cm. or more). The reason for this is given by equation (3) in paragraph 2, for the number of fringes n' between two colors λ and λ',

$$n' = \frac{e(M'\lambda - M\lambda')}{\lambda\lambda'},$$

where $M = 1 - \cos\theta$, $M' = 1 - \cos\theta'$. Since n' is a number, the glass plate can only be effective in changing θ and θ'. As both are diminished by refraction, the cosines are increased and $1 - \cos\theta$, $1 - \cos\theta'$ are both decreased. Hence n' is decreased or the number of fringes is decreased and their distance apart is thus larger.

It is obvious that when the sodium lines are not superposed, the fringes cannot lie at infinity, but are found in a special focal plane depending on the character of coincidence; *i. e.*, whether the rays are convergent or divergent. Finally a slight rotation of the slit around the axis of the collimator rotates the fringes in the opposite direction to the sodium lines and it is rather surprising, at first, that as much rotation of slit (10^0 or 20^0) is permissible, without fatally blurring the image. The slightest rotation of one grating relatively to the other destroys the fringes.

[1] About 15,000 lines to inch. Normal $\delta e \times 10^3$ = .94 cm.

[2] Focal plane changing.

3. *Homogeneous Light. Wide Slit. Transverse Axes Coincident.*— If there is no color difference, fringes of the same kind will nevertheless be seen in the telescope, on widening the slit indefinitely. Path difference is here due to differences of obliquity in the interfering rays. As in the preceding case, accurate adjustments of the longitudinal and transverse axes (in case of sodium, D_1 and D_1' or D_2 and D_2' coincide, horizontally and vertically) of the homogeneous color field are essential, if strong fringes are to appear in the principal focus. These fringes are as a rule well marked and widening the slit merely increases the width of the channelled, homogeneous field of view. If, owing to slight differences of grating space, the sodium lines are not quite superposed automatically, they may be corrected by rotating either grating, or else the apparatus as a whole, until the fringes are strongest. The fringes may be made to vanish under inverse conditions. The experiments (to be omitted here) showed their close relation to the preceding colored set, so far as motion of the micrometer is concerned. The fringes decrease in size as e increases and exhibit the same irregularity of δe values, due to the same causes (equation 6). Moreover δe was here usually found to be below the normally computed value, supposing the angle i to be negligible. In fact, Fig. 3 shows the optical center of the collimator C, so that Ca and Ca' are the axes of parallel pencils diffracted by the gratings G and G' at the angles θ for Ca and θ' for Ca'. The rays are subsequently condensed at F the focus of the telescope, L being the principal plane of the objective. The general path difference is thus for negative i, given by equation (5) which distributes the fringes from right to left with variation of i.

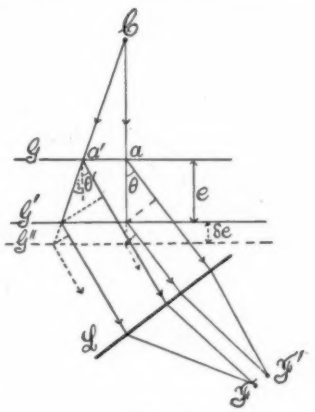

Fig. 3.

If the grating G' is displaced, δe, parallel to itself, the path difference will again be increased by λ whence

$$\delta e = \frac{\lambda \cos i}{1 - \cos(\theta' + i)}.$$

Since i is small, this equation will not differ appreciably from equation (2), with which it coincides for the central fringes.

If the sodium lines are not superposed, these fringes may still be seen; but they are not in the principal focal plane and the new focal plane

changes continually as the fringes grow in size. Experiments similar to the above bearing on the reason for the discrepancies (equation (6)) were tried with the thin Wallace gratings. As before the effect of i passing from negative (through zero) to positive values, is enormous, δe increasing nearly threefold for a change of i estimated as within 20^0.

4. *Homogeneous Light. Fine Slit. Transverse Axes Not Coincident.*— To obtain this group of interferences, the two sodium lines from a very fine slit are thrown slightly out of coincidence; *i. e.*, by not more than the D_1D_2 distance. In the principal focal plane, therefore, these doublets are seen sharply; while if the ocular is drawn sufficiently forward or rearward, an interesting class of fringes soon appears which resemble Fresnel's fringes for two virtual slits. These fringes may be seen, however, on both sides of the focal plane and increase in size with the distance of the plane of observation (focus or ocular) in front or behind the principal focal plane. In Fig. 4 the two gratings G and G' are struck by parallel pencils from the collimator at different angles of incidence (o^0 and i^0). The two diffracted pencils of parallel rays are caught by the objective L of the telescope and condensed at the principal foci F and F', appearing as two bright yellow lines. In front and behind the plane FF', therefore, are two regions of interference I and I', throughout which the Fresnellian phenomenon may be seen in any plane parallel to FF', observed by the ocular. When the electric arc is used with a very fine slit, these sodium fringes often appear at the same time as the colored fringes, and though they are usually of different sizes, their lateral displacement with a change of distance apart of the gratings, δe, is the same. The fringes in question appear alone when the sodium burner is used. They may (at times) be observed with the naked eye, with or without a lens and they then fail to appear in the telescope, unless the objective is strengthened by an additional lens. They are always vertical, but finer in proportion as the D_1D_2 and $D_1'D_2'$ doublets are moved further apart. They become infinite in size, but still strong, when the doublets all but coincide, showing a tendency to become sinuous or possibly horizontal. Rotation of either grating G around an axis normal to itself and relative to the other, produces greatly enhanced rotation of the fringes, as in all the above cases, but they soon become blurred.

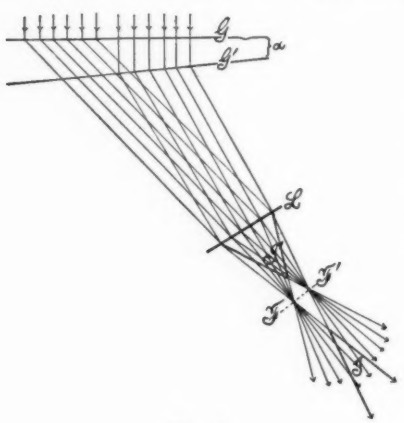

Fig. 4.

Only in the case when the horizontal axes of the field coincide (parallel rulings, etc.) do they appear strong. When the angle of incidence (or non-coincidence) is increased for both gratings, the size of the fringes increases; but when the e distance is increased by the micrometer, the fringes are apparently constant as to size. However, after displacement of 4 millimeters they are liable to become irregular and stringy, though still moving. A fine slit is not essential, particularly when e is small. They vanish gradually when the slit is too wide. If a telescope with a strong objective is used, these fringes may be seen, retaining their constant size long after those of the next paragraph vanish. Examples of data are superfluous here. With the Wallace gratings, these fringes were best produced by the aid of the sodium lines in the ordinary electric arc, simultaneously with the colored fringes and for the case of a very fine slit. They were apparent both with an ocular drawn out or drawn in. In the former case several successive groups were observed. Beginning with the sharp sodium lines in principal focus (D_2 and D_2' coincident), a slight displacement of the ocular outward showed the first group, this being a grid of very fine striations. Further displacement outward produced a second set, equally clear but larger. A third displacement of the ocular outward showed the third set and these now coincided with and moved at the same rate as the colored fringes in the same field. Other groups could not be found. No doubt for these four successive steps the interference grids of D_1 and D_1', D_2 and D_2' are coincident and superposed, until they finally find their place in the colored phenomenon.

5. *Homogeneous Light. Slit and Collimator Removed.*—Fringes similar to those seen with the wide slit above, may be observed to better advantage by removing the slit altogether. The sodium flame is then visible as a whole; and if the adjustments are perfected, it intersected with strong vertical black lines, visible to the naked eye or through a lens or a suitably strengthened telescope. They decrease rapidly with increase in e, but vanish to the eye before the preceding set (§ 4). The sodium lines need not be in adjustment, but the longitudinal axes of the field must be, as usual. If the collimator only partly fills the field of view, these diffuse light fringes and the preceding set may occur together. Both rotate markedly for slight rotation of either grating in its own plane. There seems to be a double periodicity in the yellow field, but it is too vague to be discerned. When magnified with a lens, they admit of a play of e within about .6 cm. from contact. If the sodium lines are not coincident, the focal plane continually changes with e. Otherwise it remains fixed. Again if either of the gratings is displaced and if they

are parallel, the focal plane will not change; but if G and G' are not parallel, the focal plane differs from the principal plane and now moves with the grating.

6. *Inferences.*—The above experiments showed that the equation underlying all the interferences observed is the same. These may result from different causes; but their variation in consequence of the motion of the grating, δe, is due to one and the same cause. This is best seen by producing them simultaneously in pairs. As a means of finding an accurate comparison of the number of lines per cm. on any grating, in comparison with those on the given grating, the method used in § 2 deserves consideration.

If the fringes are to be used for practical purposes, great care must be taken to keep the angle of incidence of the impinging light constant. This was not done in the present experiments, where the purpose is merely an identification of the phenomena. Moreover a micrometer with the screw running easily is essential, as otherwise the frame is liable to show appreciable twist (change of incidence) during displacement of the fringes.

The fringes are not of the sensitive type; but on the other hand they admit of a large range of displacement and are therefore adapted to special purposes.

With regard to their bearing on the behavior of reversed spectra, for the interpretation of which the present experiments were undertaken, it is obvious that the interferences with homogeneous light and a wide slit (§ 3), or in the absence of a slit (§ 5), are of analogous origin in both cases. It makes no difference, therefore, whether one of the spectra is reversed or not, except perhaps that in the former case (inversion), the coincidence of longitudinal and transverse axes is a more insistent condition. The colored fringes of § 2 obviously cannot be produced with reversed spectra. There remain the fringes with the fine slit and homogeneous light (§ 4). In other words the occurrence of a sort of generalized Fresnellian interference within the telescope, modified by causes which lie outside of it. Thus D_1 and D_1' or D_2 and D_2' may be placed sufficiently close together, to produce a region of interference before and behind the principal plane in which the sodium lines are in focus. If the D_1D_1' lines are $x = .01$ cm. apart and the fringes seen likewise at $e = .01$ cm. apart, their position measured from the principal plane will be at

$$R = \frac{xc}{\lambda} = \frac{5}{3}\text{cm.},$$

or less than 2 cm. The ocular would then have to be displaced forward

or rearward by this amount. But there are two sodium doublets, each pair of which is to interfere. Suppose that D_2 and D_1' are in coincidence so that the scheme is $D_1 : D_2D_1' : D_2'$ as in Fig. 5, where o is the principal plane of the objective and D_1 to D_2' the principal focal plane. We should then have the separate regions of interference I and I' and the combined regions I'' and I'''. When the breadth of the latter is the whole number of fringes, the two patterns clearly merge into a single pattern. The experiments show several of these stages, terminating outermost in the focal plane of the colored fringes under the given conditions. Since

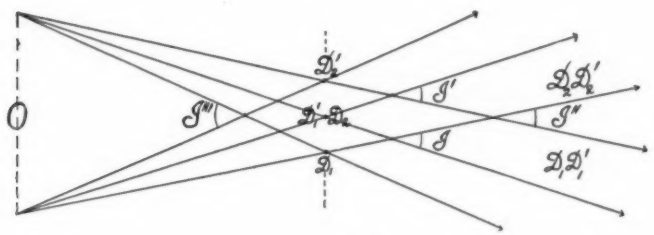

Fig. 5.

the fringes lie on hyperbolic loci, the problem itself is beyond the present purposes; but it appears that the colored fringes will not occur until the correspondence of D_1D_2 lines is shared by the whole of the two continuous spectra.

The final question at issue is the bearing of the present Fresnel phenomenon on the reversed spectra. If in Fig. 6 S and S' represent the traces of two reversed spectra in the principal focal plane, superposed

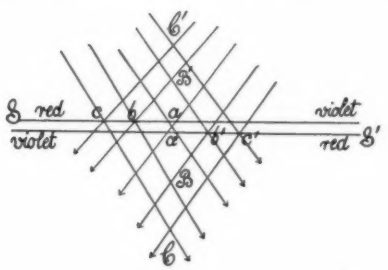

Fig. 6.

throughout their extent (*i. e.*, in longitudinal coincidence), the rays a, a', through the line of symmetry, are at once in a condition to interfere with a given difference of phase; but so are all the symmetrically placed pairs of colors c, c', b, b' of the two spectra (the distances c, c', b, b', being arbitrary), provided the corresponding rays meet. As they do not

meet in the principal focal plane, they can only interfere outside of it, b and b' at B, c and c' at C, etc. Similar conditions must hold at B' and C', inside the principal focal plane. The linear interference is thus successively transferred to different pairs of wave-lengths. The phenomena of this paper can not therefore be detected in case of reversed spectra, because in the principal focus different wave-lengths or colors are everywhere superposed, except in the narrow strip of spectrum at aa', which experiment shows to be about 1/3 the width of the sodium doublet in apparent size. The latter, however, is a question of diffraction. Beyond the principal focus, the corresponding conditions, in turn, hold for the rays at B, C, etc., B', C', etc. Hence they can not be any Fresnellian interferences (§ 4), for there are not two but only one virtual slit, as it were, and interferences are laid off in depths, along the normal $C'C$. The phenomenon may, in fact, be detected along this line for 2 or 3 meters.

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EXPERIMENTS ON THE EMISSION QUANTA OF CHARACTERISTIC X-RAYS.

BY DAVID L. WEBSTER.

INTRODUCTION.

IT has been assumed by many authors that Planck's quantum hypothesis when applied to the energy of cathode rays, would give the highest frequency of X-rays they could produce. The first accurate experimental proof of this law was given by Duane and Hunt[1] for the case of the general radiation from a tungsten target, and was extended to a cathode ray potential of 110 kv. by Hull.[2]

Off hand one would expect the law to apply to characteristic rays also. The possibility that it does not is indicated by the fact that Whiddington[3] found that the total radiation from a metal exposed to X-rays begins to increase rapidly with increasing energy of the cathode electrons when their energy is nearly 40 per cent. above that which the quantum law was later found to demand for the production of the α lines, the strongest of the K series. A similar conclusion on X-rays from a metal exposed directly to cathode rays is indicated by some later work of Beatty.[4] As the point where this occurs is not very definite and the interpretation of the results is uncertain, it seemed desirable to test each characteristic line alone with a spectrometer. A brief report of the principal results of such a test is contained in a paper which I presented to the American Physical Society in December, 1915, and published in the Proceedings of the National Academy in March, 1916. The purpose of the present paper is to report the results more fully and discuss in more detail their theoretical significance.

APPARATUS.

For this work I used a rhodium target in a tube which Dr. W. D. Coolidge, of the General Electric Company, very kindly had made up with one of his hot-wire cathodes. The spectrometer worked on the same principle as Bragg's, though the electrical system was somewhat

[1] Amer. Phys. Soc., April, 1915; PHYS. REV., Aug., 1915.
[2] Amer. Phys. Soc., Nov., 1915; PHYS. REV., Jan., 1916.
[3] Roy. Soc. Proc. Ser. A85, July 5, 1911.
[4] Roy. Soc., Proc. Ser. A 87, Dec. 13, 1912.

different. The electroscope, instead of being under the axis, was in a brass chamber, placed against the back end of the ionization chamber, whose high potential electrode was a stiff wire passing through an amber plug with a guard ring to the support of the electroscope leaf. The leaf was observed through a microscope mounted on the same arm that carried the ionization chamber. Its sensitiveness was about 10 divisions per volt at 60 volts, the potential ordinarily used; the leakage was about 1 division in 6 minutes, and the drop with a fairly strong reflection was 20 or 30 divisions per second. With reflections of less intensity its readings would repeat themselves with an accuracy of about 1 per cent.

The crystal face was a very good (1, 0, 0) face of calcite, whose irregularities were so small that they barely showed at all in photographs of the general radiation reflected by the crystal when its position was fixed and the slit was wide enough to include its whole face. The use of such a good surface made it easy to obtain comparisons of intensities in a definite narrow region of the spectrum by using the whole radiation reflected at a fixed position of the crystal with a fixed slit.

To guard against changes of the zero of angle due to heating or cooling of the support of the target, the tube was run continuously throughout each series of readings and the current was made to vary inversely with the potential. The intensities of rays given below are all reduced to a basis of 1 milliampere in the tube.

The current was furnished by 20,160 Planté cells, capable of giving 42 to 44 kv., or within 400 volts above any lower potential for which they were connected. The adjustment within this 400 volts was made by a variable water resistance. The drop in the water and battery together was kept under 1,000 volts, so that small changes in the current could produce only very small changes in the potential on the tube. This potential was measured by an electrostatic voltmeter designed by Dr. Chaffee, consisting of a small dumbbell on a bifilar suspension at ground potential, placed between two larger balls at the high potential and making an angle of 20° to 30° with their line of centers. The system was enclosed in a box lined with tinfoil and the deflection of the dumbbell was read with a telescope and scale. This voltmeter was calibrated up to 18 kv. by a manganin resistance of 894,200 ohms. Above that point the calibration was done by measuring the voltages of two parts of the battery, first separately, then in series, and then separately again. The errors in this apparatus were probably not over 1 per cent. Neighboring potentials could be compared, in any one series of measurements, to about 0.1 per cent. The current through the tube was measured with a milliammeter through which a point in the heating circuit of the

cathode was connected to the grounded end of the water resistance, the anode being connected directly to the positive pole of the battery.

The apparatus was so arranged that I could watch the milliammeter and voltmeter, and adjust the potential with the water resistance if necessary, from the start to within a few seconds of the finish of each electroscope run. Thus very little error could arise from the slow decline of voltage of the battery.

THE QUANTUM POTENTIALS OF THE CHARACTERISTIC LINES.

If the potential is gradually raised while the spectrometer is set for rays of a frequency not characteristic of the target, no rays will be detected until the potential reaches the value given by the relation

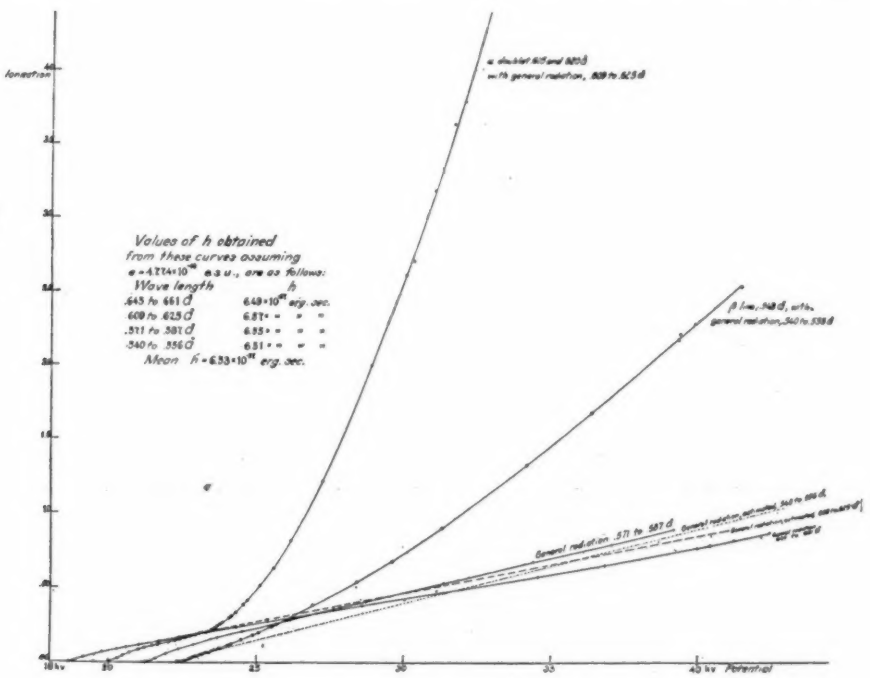

Fig. 1.

$eV = h\nu$ applied to that frequency. At this point the general radiation of that frequency will appear, and its intensity will then increase continuously with the potential. It follows that if the spectrometer is set for a characteristic frequency, it is reasonable to expect, in addition to the characteristic rays, some general radiation obeying Planck's law as at other frequencies. Whether the characteristic rays also obey it is the point to be tested here.

The results of such a test are shown by the graphs in Fig. 1. From these it is evident that the predictions about the general radiation are fulfilled; but the *characteristic rays of the α doublet appear, in addition to the general radiation, only above a definite potential about 16 per cent. higher than the quantum value. Above this point, instead of increasing nearly linearly, like the general radiation of that frequency, the characteristic rays increase at a continually accelerated rate.* Thus they begin to be a prominent part of the total radiation only at higher potentials. This explains why Whiddington and Beatty did not detect their presence in

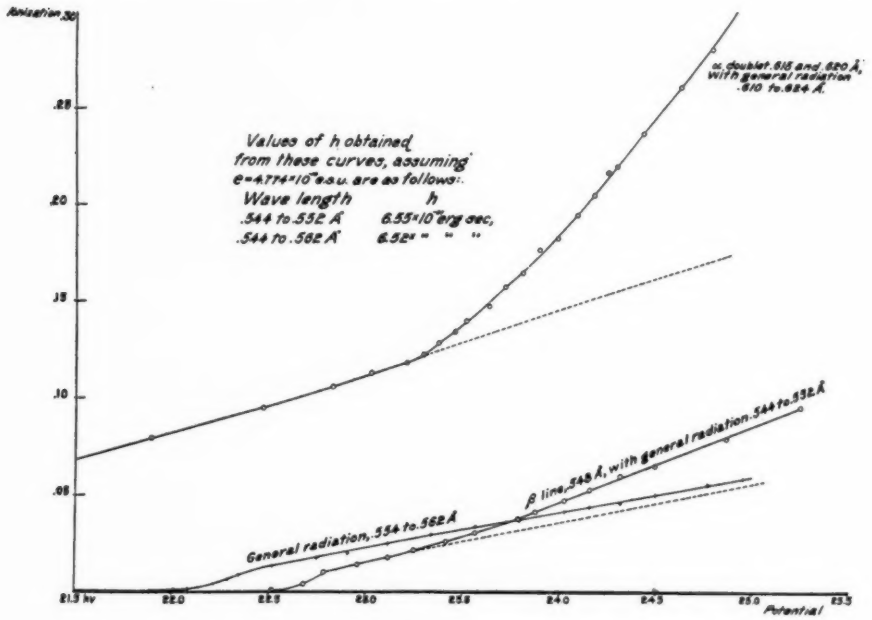

Fig. 2.

the total radiations from other metals at potentials less than 30 to 40 per cent. above the quantum value. The β graph indicates similar laws for that line, but not with any certainty when plotted on the 3cale of Fig. 1.

The question now arises, how definite is the point at which the α doublet appears? That is, does the curve shown here have a real discontinuity of curvature or of both slope and curvature, or of neither? One might presume neither, from the rounding of the bottoms of the general radiation curves, but it is easy to show that this is due chiefly, if not entirely, to the width of the slit and the consequent lack of homogeneity of the rays. The slit in this case was wide enough to include

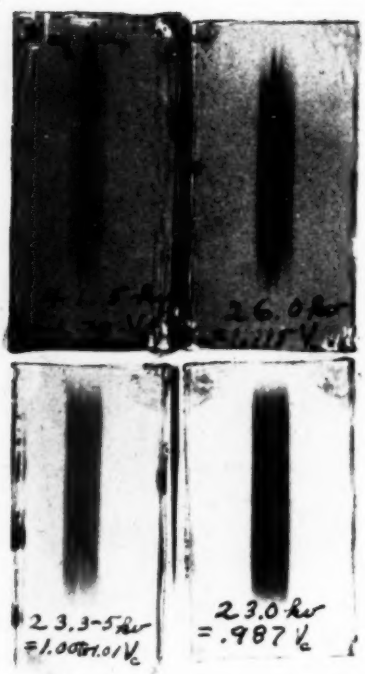

Fig. 3.

These photographs, and those of Figs. 4 and 5, have been intensified by pasting the gelatine sides of the negatives on a piece of white cardboard and photographing this combination onto a second plate. The contrast is therefore twice as good as that given by the ordinary printing process.

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the whole α doublet and a small range of general radiation on each side. This arrangement was adopted as a precaution against shifts of the focus point with potential or temperature. The boundaries of the region used are slightly indefinite, but certainly include no rays beyond the limits given with the graphs.

To obtain more evidence on the sharpness of the corner in this curve, I have taken seven series of measurements on the α doublet under good conditions, and four on the β line. One series of each kind, with a similar series for a part of the general spectrum near β, is reproduced on a larger scale in Fig. 2. If the corners in these curves are really discontinuities in either slope or curvature, it is of course impossible to prove it definitely, and all that can ever be said is that the deviations from curves with such discontinuities are less than a certain amount. With the battery and other apparatus now available such a statement can be made of discontinuities of both curvature and slope in the α curve, with limits of error of 1 per cent. in intensity and 0.1 per cent. in potential. The β line seems to have similar discontinuities, and apparently *the critical potential is the same for both lines.* As a matter of fact there are theoretical reasons, discussed below, for doubting whether the discontinuities of slope are real, and in some of the series of measurements there seem to be traces of upward curvature below the critical point, but the change of slope is certainly very sudden. The potential at which the maximum curvature of the α graph occurs is 23.3 kv. $\pm$ 1 per cent.

To prove more definitely that this is the point at which the lines first become appreciable, I have taken a series of photographs of the α doublet at different potentials, as shown in Fig. 3. These were taken with the crystal fixed and the slit open wide enough to include about .024A of the spectrum. The exposures differed in such a way as to make the background of general radiation show stronger in the low voltage pictures, although really it is less intense; but *in spite of this the lines are stronger at 41.5 kv. than at 26.0, barely visible at 23.3–5, and totally invisible at 23.0.* The steadiness of the apparatus is shown by the boundaries of the general radiation, which are as definite as the width of the source allows them to be. A motion of almost any part of the apparatus would blur them, and a motion of the crystal alone would even blur them without blurring the lines. To test the lines at a potential even nearer the critical one, I took the pictures shown in Fig. 4. For these pictures I first took a series of intensity-potential measurements near the critical point, to determine accurately its position on the voltmeter scale, then exposed at a potential 0.997 times the critical value, or only 0.08 kv. below it, then immediately repeated the intensity-potential series to be sure everything

was satisfactory, and then took the high voltage picture to be sure the lines were in the center of the field. The fact that no lines are visible at the lower voltage seems to prove that *the critical potential found by the graphs really does mark the first appearance of any appreciable quantity of characteristic rays.*

COMPARISON OF LINES ABOVE THE CRITICAL POTENTIAL.

This point being settled, the question arises whether the characteristic lines, which all appear at the same potential, show any similarity of behavior above it. The photographs in Figs. 3 and 4 show no obvious difference in the behavior of α_1 and α_2, and even those of Fig. 5 show no certainty of change in their ratio. Such a test is necessarily very inaccurate, especially as the presence of different amounts of general radiation impairs the accuracy of both the eye and the plate. Nevertheless, when one remembers that the exposure at 39kv. was only 8 minutes and at 23.4 kv. 16 hours, or nearly 1,000 minutes, and that the current varied inversely as the potential, it appears that the α_2 line is about 200 times as strong in the former picture as the line and the general radiation back of it in the latter. The lines alone, according to the ionization measurements, must be well over 1,000 times as strong in the former as in the latter. Thus even such a rough test as this acquires some value.

The relation between the α doublet as a whole and the β line is obtainable from the graphs in Fig. 1, where we may extrapolate for the amount of general radiation with each line as shown in the dotted curves. The points where these cross the other general radiation curves may be obtained approximately from the fact that such a crossing signifies a maximum of intensity in the spectrum about half way between the wave-lengths concerned, and that this maximum shifts steadily to the shorter wave-lengths with increasing potential. The curves thus obtained have the right slopes at the critical potential and are confirmed within the limits of error by plotting the spectra at different potentials, as shown below. Subtracting the estimated general radiations from the total intensities, one obtains the curves given in Fig. 6. The ratio α/β is indeterminate at the critical potential and very inaccurate near it, but soon becomes constant. In fact, *if the ordinates of the α curve are divided by this constant and plotted, they all lie on the β curve, almost within the width of the line used to represent it.* The absolute value of this ratio is of little importance, since these "intensities" being ionization currents in an arbitrary amount of ethyl bromide, are on slightly different scales, but the constancy of this ratio seems very significant.

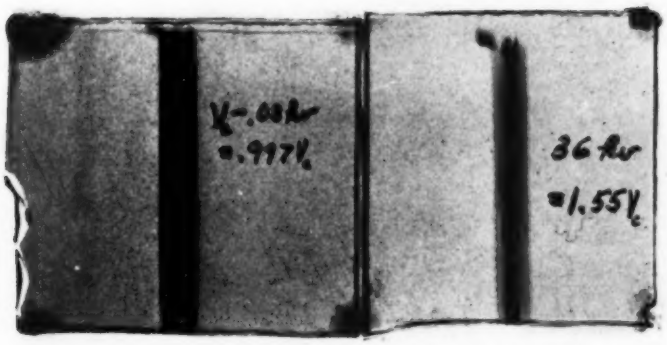

Fig. 4.

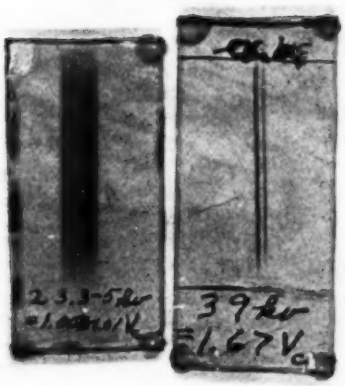

Fig. 5.

Exposure at 39 kv., 8 minutes; at 23.3–5 kv., 16 hours.

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To test this point by an independent method, I have plotted the spectra shown in Fig. 7. This test is really not so conclusive as the other, because of the long time necessary for determining one of these spectra, and also because of the uncertainty, with the narrow slit used in such measurements, of locating and measuring the exact maximum of each line. The ratios in which the lines increase between 31.8 and 40.0 kv. are nevertheless nearly equal, and the slight inequality is in the direction

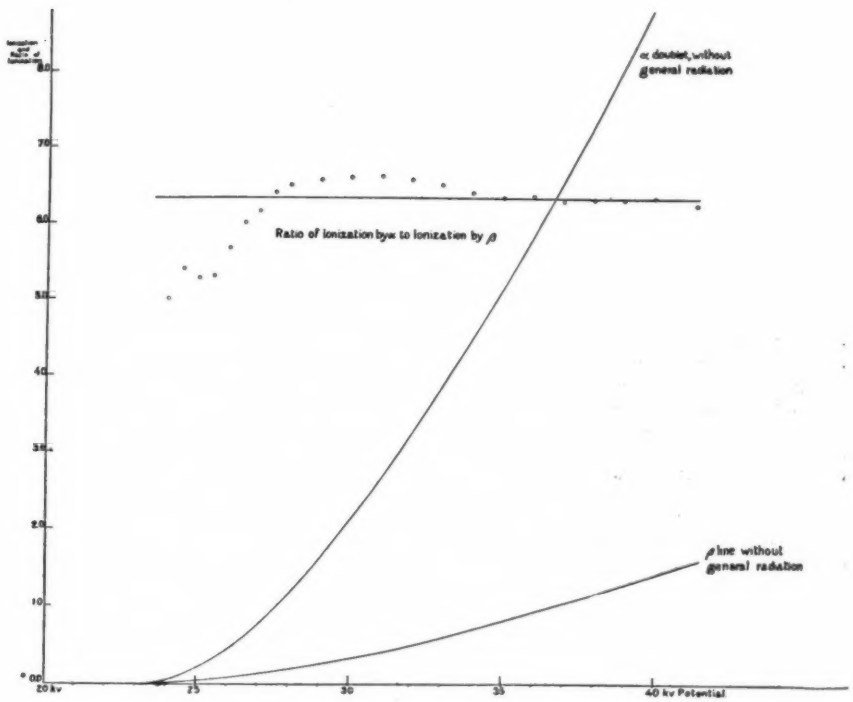

Fig. 6.

opposite to that appearing in the other method. In addition to confirming the previous results these curves show that the γ line also increases in the same ratio as the others. For data on the α_1 line we can take the height of the lower hump in the α curve. The ratio of its intensities seems to be between 2.8 and 3.3, agreeing with the others as well as one can expect of such an uncertain measurement. Hence we may state as a second principle resulting from these experiments that *above the critical potential the lines all increase in the same ratio for any given increase in potential.*

THE CRITICAL WAVE-LENGTH.

These spectra also give some information on other points, such as the way a small impurity of ruthenium in the target produces its lines as if the rhodium were not there, exactly as the zinc showed its presence in a brass target used by Moseley in his well-known series of wave-length measurements.

Another interesting point shown by these spectra is the series of values

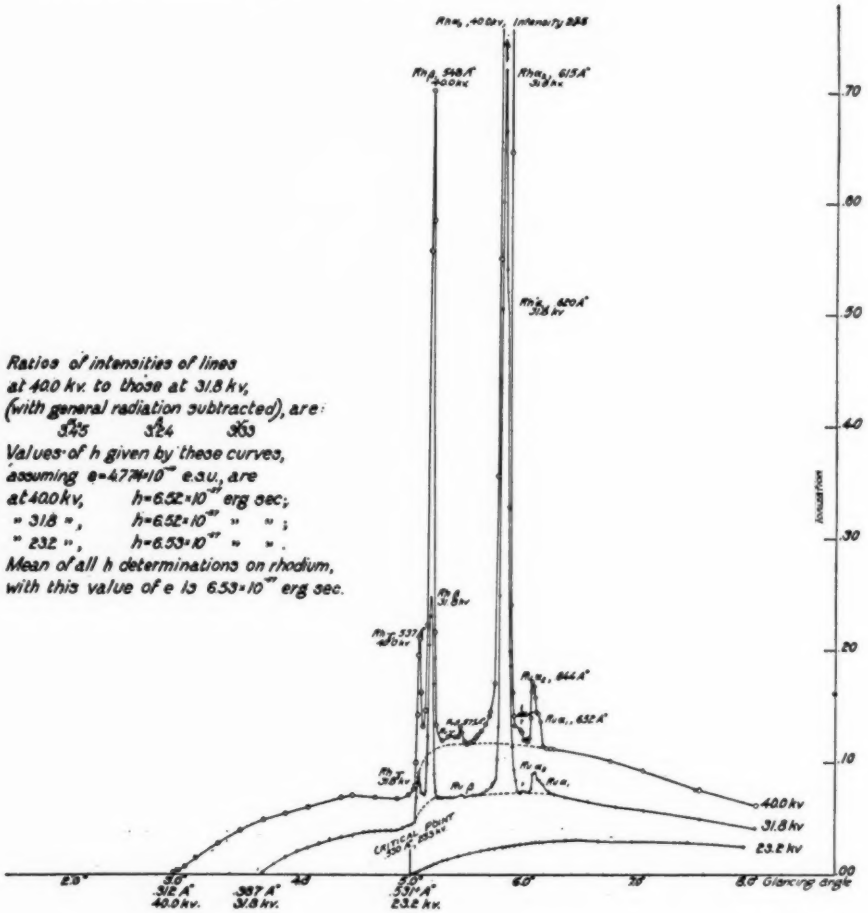
Ratios of intensities of lines
at 40.0 kv. to those at 31.8 kv,
(with general radiation subtracted), are:
3.45 3.24 3.69
Values of h given by these curves,
assuming e=4.774×10⁻¹⁰ e.s.u., are
at 40.0 kv, h=6.52×10⁻²⁷ erg sec;
" 31.8 ", h=6.52×10⁻²⁷ " " ;
" 23.2 ", h=6.53×10⁻²⁷ " " .
Mean of all h determinations on rhodium,
with this value of e is 6.53×10⁻²⁷ erg sec.
40.0 kv
31.8 kv
23.2 kv
Glancing angle

Fig. 7.

of h obtained from their short-wave limits. The agreement between these values is much better than was expected, and they also agree surprisingly well with the mean of the values from the intensity-potential curves.

For all the wave-lengths and h values given here, Bragg's grating space for calcite was recomputed on the basis: $e = 4.774 \times 10^{-10}$ esu. (Milli-

kan), atomic weight of silver = 107.88, electrochemical equivalent of silver = .00111827 gm. per co., molecular weight of calcite = 100.09, $c = 2.9986 \times 10^{10}$ cm. per sec. (Last four from Kaye and Laby's tables.) These data raise the grating spacing from 3.025Å to 3.03, and give for the mean value of h, 6.53×10^{-27} erg. sec. This lies above the values given by Duane and Hunt[1] and Planck,[2] and below those given by Millikan and Hull.[3]

The most important information given by these spectra is the location of the wave-length whose quantum potential is the critical value for the production of the α and β lines. For this wave-length, .530Å, is between 1.0 and 1.5 per cent. short of the γ line, and is in just the part of the spectrum where the absorptive power of the rhodium increases suddenly with decreasing wave-length. Some evidence of this increase of absorption is given here by the drop of intensity of the general radiation. For this drop is undoubtedly due to the slight roughening of the surface of the target by the cathode rays, combined with the fact that the rays leave it at a very small angle to the surface and therefore the humps must screen off the more easily absorbed rays coming from the hollows. In fact Hull[4] has shown that in the cases of tungsten and molybdenum the spectrum without such screening shows neither a rise nor a drop near the γ line, but with a thin screen of the same material as the target the spectrum is deformed in exactly this way. As a matter of fact I find that the point where this occurs seems to be at the γ line itself, rather than the critical wave-length found by potentials, but as I shall point out below, the latter is probably too short. I have also obtained several photographs which show that this change does occur at the γ line; but these are not suitable for reproduction, and an accident to the apparatus has rendered further work impossible for the present.[5]

We may now state the results of the present experiment in the form: *The K series rays from a rhodium target are emitted in appreciable quantities only if the potential is great enough to excite general radiation of a wave-*

[1] Amer. Phys. Soc., April., 1915; PHYS. REV., Aug., 1915. Recomputed on the basis used here, their value is 6.50.

[2] "Heat Radiation," Translation by Masius, p. 172, gives 6.415×10^{-27} erg sec. for h and 4.67×10^{-10} esu. for e.

[3] Amer. Phys. Soc., Dec., 1915. Their values are 6.57 and 6.59 respectively.

[4] L.c., note 7.

[5] Note added in proofs. Since this was written I have got the apparatus working again, and have obtained a better series of photographs of the γ line and its neighborhood, with the tube turned at different angles to the direction of the rays used. These also show a drop of intensity of the general radiation on passing the γ line when the angle of the rays from the target is small, but none when it is as large as 10°. This confirms the above statement of the origin of the drop and the location of the discontinuity in the absorption spectrum at the γ line itself.

length slightly less than that of the γ line, at which the sudden change of absorptive power occurs. Since the K series lines in all elements are very similar, the law in this form probably holds for them all. *Above this potential the K series lines all increase in the same ratio for any given increase of potential; their rates of increase are continually accelerated, while that of any short region in the general spectrum is at first retarded and then nearly constant. Taking e as 4.774×10^{-10} esu, Planck's h appears to be 6.53×10^{-27} erg. sec.*

THE BASIS FOR INTERPRETATION OF THESE RESULTS.

The significance of these results depends of course on what are the fundamental laws of X-ray phenomena. If these are the laws of electromagnetics as applied to wireless telegraph radiation, we can draw one set of conclusions; if they are laws which would permit the existence of such an atom as Bohr's, our conclusions must be entirely different.

As Hull has pointed out,[1] the critical potential found in these experiments is very consistent with what might be predicted from Kossel's model of the atom,[2] which contains revolving rings of electrons rather similar to those of Bohr, and determines the frequencies of radiations by the quantum relation applied to the energy of an electron dropping from one ring to another. Such an atom, nevertheless, is known to present many difficulties, such as that of accounting for the stability of the rings in the presence of one another, and even more in the presence of other atoms. Another objection, often overlooked, is that such an atom, even though it does contain the quantum relation, does not satisfy the condition that *an oscillator must be able to possess any number of quanta at one time* without affecting its frequency. Since this is one of the essential conditions of Planck's development of his law[3] the failure to satisfy it deprives such an atom of one of the chief advantages of the introduction of the quantum.

Moreover there is the difficulty of accounting for the failure of the rings to lose energy rapidly by radiation, especially now that we have the evidence of X-ray reflection as well as optical phenomena in favor of the electromagnetic wave theory. The argument against the wave theory based on quantum relations does not seem valid, since I have proved in a previous paper[4] that these relations can be reconciled with

[1] Amer. Phys. Soc., Dec., 1915.

[2] Verh. d. D. Phys. Ges., Nov. 30, 1914; see also Wagner, Ann. der Phys., 1915, and Cabrera, Nature, Oct. 7, 1915.

[3] L. c., p. 153.

[4] Amer. Acad. Proc., Jan., 1915.

a form of the classical electromagnetic theory containing a few additions suggested by Parson's magneton theory of the atom,[1] and Planck's method of deriving his radiation law. Consequently the laws of the classical electromagnetics will be taken as a basis for the present discussion. I shall not attempt to give a complete picture of the X-ray mechanism, or even state whether the emitting oscillators are in the nucleus of the atom or outside it, but shall merely attempt to show what conclusions about this mechanism can be drawn from the experiments described above.

THE RELATION BETWEEN THE GENERAL AND CHARACTERISTIC RAYS.

Proceeding along this line, one point may be deduced from the fact that there is a definite high-frequency limit to an X-ray spectrum at a constant potential. By Fourier's series or by direct combination of reflected waves from individual layers of atoms in the crystal, it may readily be shown[2] that this fact is inconsistent with the pulse theory of X-rays and signifies that *X-rays of the general spectrum are emitted in periodic trains by oscillators of definite frequencies, even though the spectrum is continuous.* Of course there is bound to be a certain amount of irregular, or pulse, radiation, which is impossible to calculate *a priori*, but which is thus proved to be extremely small.

We can even go a step further and say that the fact that the end of the spectrum can be located within 1 per cent. proves that there must be at least 100 waves in the train[3] to give the necessary interference. Probably there are many more.

The existence of oscillators of frequencies distributed throughout the spectrum is indicated also by the evidence of absorption. For the gradual decrease of absorption with increasing frequency, interrupted by a sudden increase near each characteristic series, is not explainable on a basis of oscillators of any one or small number of frequencies, least of all can it be attributed to the characteristic ones.[4] Consequently it is not surprising that the evidence of the emission spectra also indicates the existence of all frequencies in the target, although not necessarily all in one atom at one time.

[1] A. L. Parson, "A Magneton Theory of Atomic Structure," Smithsonian Miscellaneous Collections, Vol. 23, No. 11, 1915.

[2] D. L. Webster, Amer. Phys. Soc., April, 1915; PHYS. REV., July, 1915.

[3] From a theory of reflection which I published in the PHYSICAL REVIEW in March, 1915, it might appear as if we could find an upper limit to the length of these trains from the depth of penetration of the rays in the crystal. However, other considerations, noted under the heading, "Errata and Addenda," PHYSICAL REVIEW, June, 1916, have shown that the main hypothesis of that theory must be incorrect, and therefore that the effects it was designed to explain must be due to other causes than the shortness of wave trains assumed there.

[4] See *e. g.*, Lorentz, Theory of Electrons, p. 154.

In view of this agreement, it is surprising that where the absorption suddenly increases, near the γ line, there is no simultaneous increase of emissivity.[1] This may be explained by an hypothesis which we shall find extremely useful, namely that *the K characteristic rays produced by the action of cathode rays are due to the same mechanism as those produced by fluorescence.*

The natural assumption that they are produced as true fluorescence by absorption of the higher frequency rays entering the target is precluded by the experiments of Beatty[2] on total radiation. However, when an oscillator does produce the K fluorescence, it must obtain the energy by absorption from higher frequency X-rays and store it in some non-radiating system such as the magneton theory[3] provides, until it has acquired a whole quantum. Then it must remit at least a part of this quantum in a complex vibration whose component frequencies are those of the K lines. The quantum relation would indicate that if its absorbing frequency is not much above that of Kγ it probably emits most of its energy in this way. Now if this same mechanism acquires a quantum from a cathode particle, the hypothesis given above indicates that it emits most of this energy likewise as K radiation. Hence the K series oscillators will have little effect on the general radiation near the K lines, and this general radiation must be due to oscillators associated with the L, M, etc., series. Under these circumstances we should expect a lack of any sudden rise of emissivity at the critical frequency of the absorption spectrum, just as it is observed.

The Explanation of the Intensity Relations of the Lines.

The same hypothesis on the origin of the K rays explains at once why a quantum large enough for one of the higher frequency oscillators is required for the production of any characteristic rays. Likewise it explains the contrast between the upward curvature of their intensity-potential curves and the linearity which I have found to hold for those of the general radiation above this critical frequency as well as below it. For as the potential is raised it increases not only the number of oscillators of a given frequency which a cathode particle can reach and excite, but also the range of frequency of the oscillators for which it has a quantum. Consequently, since each of these factors increases more or less linearly, their product increases as something like the square of either. Thus the intensity-potential curve for the characteristic rays, due to the excitation of such oscillators, should be exactly what is observed.

[1] Cf. Hull. l. c., note 7.
[2] L. c., note 4.
[3] L. c., notes 13 and 14.

Since energy is required for the increased emission of high-speed electrons accompanying the fluorescent K radiation, it might be supposed necessary to allow for this energy in these comparisons. The most conclusive argument against this supposition is perhaps the work of Barkla and Shearer,[1] who have found that these electrons have the same energy as those associated with L radiation. Consequently each electron requires a whole quantum, leaving nothing for secondary X-rays from the atom emitting it. This indicates that the oscillator can emit either an electron or a quantum of X-rays, but not both at the same time. The present discussion is confined to those emitting X-rays only.

If the absorptive power has a finite discontinuity at the critical frequency, as it seems to, it is easy to prove that the present hypothesis leads to finite discontinuities in the curvatures of the characteristic intensity-potential graphs, but none in their slopes. Thus the apparent difference between the wave-length where the absorption changes and that whose quantum potential is the potential of maximum curvature of these graphs is explained as a result of the inability of the apparatus to detect the extremely small amounts of characteristic radiation in the presence of the general at potentials very near the actual critical value. *Probably some characteristic rays are produced at any potential above the quantum value for the γ frequency, though for the first one or two hundred volts the amount must be very small indeed.*

Another experimental fact to be explained is the constancy of the ratio of intensities of any two lines, a state of affairs remarkable for its difference from that of lines of the same series in light. Of course no atom can hold an infinite number of oscillators at one time, as it would have to to produce the whole general spectrum alone. Moreover, the absence of selective absorption of the characteristic rays by the element producing them shows that when the oscillator finishes emitting a set of these rays its frequency must rise again into the region of stronger absorption. Presumably it is a matter of chance what higher frequency it assumes, since any predetermined frequency for an atom would mean a difference between it and other atoms of the same element with different predetermined frequencies. Thus we must conclude that *the mechanism emitting characteristic rays is not only the same in the case of direct production as in fluorescence, but also the same for fluorescence from all higher frequencies and direct production with all sufficient potentials.* Thus it is evident that we should expect the complex characteristic vibration to be the same and the ratios of its component lines to be the same, in all cases, just as they are found to be.

[1] Phil. Mag., Dec., 1915.

SOME EVIDENCE ON THE MECHANISM OF ABSORPTION.

These data and conclusions are far from being enough for a complete picture of the X-ray oscillator. A mystery which should not be overlooked arises from the fact that if an oscillator has stored up even a large fraction of a quantum from absorbed X-rays it still cannot be made to radiate by a cathode particle having less than a whole quantum, even though the sum of its energy and that of the cathode particle might be almost twice that amount. This fact indeed seems fatal to any model of the oscillator in which the energy absorbed from X-rays is stored as vibratory motion of any particle with which a cathode electron might collide. According to the magneton theory, the stored energy is in the form of a slight increase in the velocity of circulation of the current in the magneton, produced gradually by a mechanism which damps the oscillations on it. There is no obvious way in which a colliding particle could influence this store of energy appreciably, though it might produce a large displacement of the ring and the charge upon it. If this theory is correct, the phenomenon under consideration would indicate that the collision must be elastic unless the energy of these displacements by themselves amounts to a quantum, in which case it may be radiated, rather than returned to the colliding electron.

This assumption is very dubious, being based on but one phenomenon and not as yet subject to test by others. But even if the magneton theory turns out to be incorrect, the facts seem to show clearly an independence of the cathode ray and stored X-ray energies that must have some such counterpart in the modifications they produce in the system.

SUMMARY.

The chief experimental facts proved here are:

1. Rays of the K series of rhodium are produced only at potentials above that required to excite general radiation of a frequency equal to or slightly greater than that of the γ line.

2. Above this potential all the K lines increase in the same ratio for any given increase of potential; their rates of increase are continually accelerated, while that of any short interval in the general spectrum is at first retarded and then nearly constant.

3. The value of Planck's h appears to be 6.53×10^{-27} erg sec. taking e as 4.774×10^{-10} esu.

In the discussion of these results it is assumed that X-rays must obey the laws of the classical electromagnetics and that the quantum relations must be due to an ability of the oscillator to store absorbed

energy in some non-radiating system such as the circulation postulated in Parson's magneton theory. On this basis it is shown that:

1. The short-wave limit of the spectrum at a constant potential indicates that even the general radiation consists of wave trains and not of pulses, and therefore that X-rays are emitted by oscillators of definite frequencies distributed throughout the spectrum; this conclusion is strengthened by evidence from the absorption spectrum.

2. All the behavior of the K-rays described above is explainable on an assumption suggested by the general emission and absorption spectra, namely that the same mechanism which emits fluorescent characteristic rays also emits those which arise directly from the impact of cathode rays without true fluorescence.

3. The quantum law for the end of the spectrum indicates that the energy of cathode particle cannot combine with that already stored to produce a quantum; this appears inconsistent with any type of atom in which the energy is stored as vibratory motion, but not necessarily with Parson's theory of the atom.

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NOTE ON THE END EFFECT IN THE ELECTROSTRICTION OF CYLINDRICAL CONDENSERS.

BY EDWIN C. KEMBLE.

THERE are two methods of computing the deformations of bodies due to the action of electrostatic fields. One method substitutes the expressions for the ponderomotive body and surface forces derived in the general theory of electrostriction into the equations of elastic equilibrium and attempts to solve those equations subject to boundary conditions appropriate to the body under consideration. This will be called the "elasticity" method. The second mode of attack makes no use of the general theory but refers each particular problem back to fundamental principles. It assumes that the electrical and elastic state of the body can be defined with sufficient exactness by means of a finite number of coördinates. (For example, in the problem of the elongation of a charged cylindrical condenser, the coördinates chosen are the difference of potential between the coatings, V, and a longitudinal traction, q, to which the cylinder is supposed subjected.) Otherwise the assumptions of the two methods are the same. The second is essentially an energy method and will be referred to as such.

Hitherto the application of these two methods to the problem of the elongation of a charged cylindrical condenser has led to contradictory formulas. Sacerdote,[1] using the energy method, derived a formula for the elongation of a condenser with adherent coatings in 1899 and one for the elongation of a condenser with nonadherent coatings in 1901. The first of these formulas did not seem to be in accord with the experimental results of More[2] and of Shearer.[3] These experimenters found no elongation that could not be accounted for as the result of unavoidable heating, although the sensitiveness of their apparatus seemed sufficient to detect elongations of the order of magnitude of that predicted by the Sacerdote formula. This apparent disagreement between theory and experiment led Adams[4] to attack the theoretical problem by the elasticity method. In 1911 he published formulas derived in this way which differed from the Sacerdote formulas and seemed to be in better accord

[1] Sacerdote, Journal de Phys. (3), t. VIII., p. 457, 1899; t. X., p. 196, 1901.

[2] L. T. More, Phil. Mag. (5), I., p. 198, 1900; (6), VI., p. 1, 1903; (6), X., p. 676, 1905.

[3] J. S. Shearer, PHYS. REV. (1), XIV., p. 89, 1902.

[4] E. P. Adams, Phil. Mag. (6), XXII., p. 889, 1911.

with the experimental work. The present paper proposes to show that the formulas derived by the two different methods may be brought into agreement if the action of certain longitudinal forces due to the electric field in the neighborhood of the ends of the armature coatings be taken into account.[1] It will also show that the negative results of More and of Shearer are not to be interpreted as a proof that there is no such thing as electrostriction.

The capacity of a cylindrical condenser of finite length is given approximately by the formula for the capacity of a section of an infinite cylindrical condenser having the same dimensions. To get the exact formula for the finite cylindrical condenser we must add to the approximate capacity an end correction independent of the length. This end correction becomes negligible in comparison with the whole capacity if the cylinder is very long. Reasoning by analogy one would be led to suppose that the end effect on the electrostriction would be negligible in the case of a long cylinder. But this is not the case. The longitudinal traction exerted by the electric field at the ends of the coatings is independent of the length of the coatings and consequently the elongation produced is proportional to that length.

We adopt the following notation:

x, y, z, Cartesian coördinates,
r, θ, z, cylindrical coördinates,
$\mathfrak{E}$, (Components $\mathfrak{E}_x$, $\mathfrak{E}_y$, $\mathfrak{E}_z$) electromotive force,
$\mathfrak{D}$, (Components $\mathfrak{D}_x$, $\mathfrak{D}_y$, $\mathfrak{D}_z$) electric displacement,
F, (Components F_x, F_y, F_z) ponderomotive force per unit volume,
f, (Components f_x, f_y, f_z) ponderomotive force per unit area,
K, specific inductive capacity,
E, Young's modulus,
σ, Poisson's ratio,
a, internal radius of dielectric tube,
b, external radius of dielectric tube,
d, thickness of dielectric tube $(b - a)$,
L, length of coatings,
V, difference of potential between coatings,
ρ, volume density of electric charge,
δ_1, δ_2, constants characterizing the change in the specific inductive capacity of the dielectric when it is strained.

The general theory of electrostriction gives the following expression

[1] Professor Adams has called my attention to the review of his article in the Beiblätter to the Annalen der Physik in which Pockels remarks that the discrepancy is probably due to the neglect of an end effect. So far as the present writer is aware this is the only published criticism of the work of Professor Adams.

for the x-component of the ponderomotive force per unit volume in a dielectric which is isotropic at every point but not necessarily homogeneous:

$$F_x = \rho\epsilon_x - \frac{\epsilon^2}{8\pi}\frac{\partial K}{\partial x} - \frac{1}{8\pi}\left\{\frac{\partial}{\partial x}\left[\mathfrak{E}_x^2\delta_1 + (\mathfrak{E}_y^2 + \mathfrak{E}_z^2)\delta_2\right] + \frac{\partial}{\partial y}\left[\mathfrak{E}_x\mathfrak{E}_y(\delta_1 - \delta_2)\right] + \frac{\partial}{\partial z}\left[\mathfrak{E}_x\mathfrak{E}_z(\delta_1 - \delta_2)\right]\right\}. \quad (1)^1$$

The y- and z-components may be obtained by cyclic advancement. In the case of an uncharged homogeneous dielectric the above reduces to the simple vector expression

$$F = -\frac{(\delta_1 + \delta_2)}{16\pi}\,\text{grad}\,(\mathfrak{E}^2). \quad (2)^2$$

By means of (1) and its companion equations the expanded Maxwell "fictitious" stress system

$$P_{xx} = \frac{K}{8\pi}(\mathfrak{E}_x^2 - \mathfrak{E}_y^2 - \mathfrak{E}_z^2) - \frac{1}{8\pi}(\mathfrak{E}_x^2\delta_1 + \mathfrak{E}_y^2\delta_2 + \mathfrak{E}_z^2\delta_2),$$
$$\cdots \quad (3)^3$$
$$P_{yz} = \frac{K}{4\pi}\mathfrak{E}_y\mathfrak{E}_z + \frac{1}{8\pi}(\delta_2 - \delta_1)\mathfrak{E}_y\mathfrak{E}_z$$
$$\cdots$$

may be obtained. The expressions for the force exerted at a surface of discontinuity may be obtained either from (1) directly or from (3). Suppose that the x-axis coincides with the normal drawn from the medium A to the medium B. Then the components of the force per unit area exerted at the surface are

$$f_x = 2\pi\mathfrak{D}_x^2\left(\frac{1}{K''} - \frac{1}{K'} - \frac{\delta_1''}{K''^2} + \frac{\delta_1'}{K'^2}\right) - \frac{(\mathfrak{E}_y^2 + \mathfrak{E}_z^2)}{8\pi}(K'' - K' + \delta_2'' - \delta_2'),$$
$$f_y = \frac{\mathfrak{E}_y\mathfrak{D}_x}{2}\left(\frac{\delta_2'' - \delta_1''}{K''} - \frac{\delta_2' - \delta_1'}{K'}\right), \quad (4)^4$$
$$f_z = \frac{\mathfrak{E}_z\mathfrak{D}_x}{2}\left(\frac{\delta_2'' - \delta_1''}{K''} - \frac{\delta_2' - \delta_1'}{K'}\right),$$

where primes refer to the medium A and double primes to medium B.

[1] This is a specialization of the formulas given by Pockels, Encyk. der Math. Wiss., Band V$_2$, Heft 2, pp. 354 and 360.

[2] Pockels, l. c., p. 361.

[3] Pockels, l. c., equations (5) and (11′).

[4] Pockels, l. c., equation (13).

Adherent Armatures.

Now consider the case of a cylindrical condenser with adherent armatures. Let the z-axis coincide with the axis of the cylinder. Then the electric force at points far from the ends of the coatings is

$$\mathfrak{E}_0 = \frac{V}{r}\frac{1}{\log b/a} \equiv \frac{S}{r}. \tag{5}$$

Substituting this value of $\mathfrak{E}$ into (2) we see that the ponderomotive force is radial and has the value

$$F_r = \frac{(\delta_1 + \delta_2)}{8\pi}\frac{S^2}{r^3}. \tag{6}$$

The surface forces are normal to the coatings and have the values

$$\begin{aligned} f_r \text{ (outer surface)} &= -\frac{(K - \delta_1)}{8\pi b^2} S^2, \\ f_r \text{ (inner surface)} &= +\frac{(K - \delta_1)}{8\pi a^2} S^2. \end{aligned} \tag{7}$$

Professor Adams assumes that the longitudinal stress is zero for an open tube and computes the elongation due to the forces specified on the assumption that the cylinder is thin. He obtains

$$\Delta_1 L = -\left(\frac{V}{d}\right)^2 \frac{(\delta_1 + \delta_2)\sigma L}{8\pi E}. \tag{8}$$[1]

The formula of Sacerdote is

$$\Delta L = \left(\frac{V}{d}\right)^2 \frac{[K + \delta_2 - (\delta_1 + \delta_2)\sigma]L}{8\pi E}. \tag{9}$$[2]

Both Adams and Sacerdote assume that the armature coatings are so thin that their tensile strength may be neglected.

Let us now compute the electrical ponderomotive forces exerted at the ends of the coatings and add the elongation which they produce to that already computed on the assumption that there is no longitudinal traction.

There are three possible sources of longitudinal stress.

(*a*) At the edges of the coatings there is a strong electric force directed along the axis of the cylinder. The intensity of this force is evidenced by the brush discharge which frequently takes place at the edges of the coatings when they are charged to high potentials. This strong electric

[1] E. P. Adams, Phil. Mag., 6, XXII., p. 892, equation (12), neglecting $\left(\frac{d}{2a}\right)\left(\frac{1}{1-\sigma}\right)$ in comparison with unity.

[2] E. P. Adams, l. c., equation (15); Pockels, l. c., equation (23).

field is accompanied by a mechanical force tending to stretch the coatings. This force is applied in the first place to the coatings but the stress is at once transmitted by them to the cylinder of dielectric.

(*b*) At points far from the ends of the cylinder the electric force in the dielectric is a function of r only, so that the body force, $-\frac{(\delta_1 + \delta_2)}{16\pi}$ grad $\mathfrak{E}^2$, has no component along the z-axis. But near the edges of the coatings the value of $\mathfrak{E}^2$ begins to diminish and drops nearly to zero a short distance beyond the ends of the armatures. In this region the force

$$F_z = -\frac{(\delta_1 + \delta_2)}{16\pi}\frac{\partial(\mathfrak{E}^2)}{\partial z}$$

must be taken into account.

(*c*) The third source of longitudinal traction is the z-component of the force on that part of the surface of the cylinder which is not covered by the armature coatings. (Where the coating is in contact with the surface of the dielectric the ponderomotive surface force has no tangential component.)

For our present purpose it is not necessary to compute these forces separately. The resultant of all three may be obtained very simply from the expanded Maxwell stress system. The fundamental property of this system is that the surface integral of the stress over any closed surface is equal to the resultant ponderomotive force exerted on the material inside the surface. Consequently the total longitudinal pull exerted by the electric field in the neighborhood of the ends of the coatings may be obtained by integrating the z-component of the Maxwell stresses over a surface which encloses one end of the coatings.

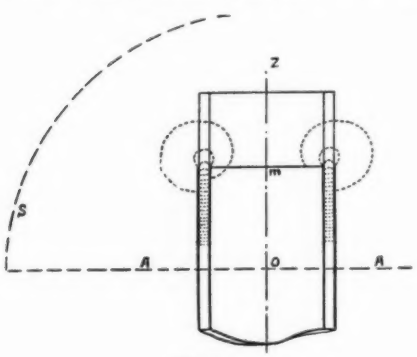

Fig. 1.

To make the computation as simple as possible we choose as our surface, S (Fig. 1), a hemisphere with its base plane A–A normal to the axis of the cylinder and cutting that axis at a point far from either end of the armatures. In the limit when the radius of the hemisphere increases indefinitely the integral of the Maxwell stress over S approaches the integral of the Maxwell stress over the plane A–A. Let S_1 denote the annular section in which the plane A–A cuts the cylinder. Let S_2 denote the rest of the plane A–A. As the distance $\overline{om}$ from the plane

A–A to the end of the coatings is indefinitely increased the integral of the Maxwell stress over S_2 will approach zero as a limit. Let P denote the longitudinal traction to be computed. Then to a close approximation P is equal to the integral of the z-component of the Maxwell stress over S_1 only.

$$P = \int\int_{S_1} P_{zz} ds = \int\int_{S_1} \frac{\mathfrak{F}^2(K + \delta_2)}{8\pi} ds = \frac{K + \delta_2}{4} \int_a^b \mathfrak{F}^2 r dr.$$

But

$$\mathfrak{F}^2 = \left(\frac{V}{r}\right)^2 \left(\frac{1}{\log b/a}\right)^2.$$

Hence

$$P = \frac{V^2(K + \delta_2)}{4(\log b/a)^2} \int_a^b \frac{dr}{r} = \frac{V^2(K + \delta_2)}{4 \log b/a}. \tag{10}$$

Now our preliminary analysis of the sources of this traction shows that the electrical field exerts no longitudinal pull except near the ends of the coatings. Hence we can obtain the elongation produced to a very close approximation if we treat P as a force applied directly to the ends of the coatings. Then

$$\frac{\Delta_2 L}{L} = \frac{P}{E\pi(b^2 - a^2)} = \frac{V^2(K + \delta_2)}{4\pi E(b^2 - a^2) \log b/a}.$$

If $(b - a)$ is small we have approximately

$$(b^2 - a^2) \log b/a = 2d^2.$$

Hence

$$\frac{\Delta_2 L}{L} = \frac{(K + \delta_2)}{8\pi E} \left(\frac{V}{d}\right)^2. \tag{12}$$

Adding this elongation to that obtained by Professor Adams we have

$$\frac{\Delta L}{L} = \frac{\Delta_1 L}{L} + \frac{\Delta_2 L}{L} = \frac{1}{8\pi E} \left(\frac{V}{d}\right)^2 [K + \delta_2 - (\delta_1 + \delta_2)\sigma], \tag{13}$$

which is the formula given by the energy method.

NON-ADHERENT ARMATURES.

Let us now suppose that the armatures are not in contact with the cylinder of dielectric but are separated from it by a nonconducting intermediary fluid of dielectric constant K'. Let g and f denote the radii of the outer and inner armatures respectively. The electric force at points in the interior of the dielectric and far from the ends of the condenser will be sensibly uniform. Denote its value by H.

The formula given by Sacerdote for this case is

$$\frac{\Delta L}{L} = \frac{H^2}{8\pi E}[\delta_2 - (\delta_1 + \delta_2)\sigma]. \tag{14}$$[1]

Professor Adams obtains

$$\frac{\Delta_1 L}{L} = -\frac{H^2}{8\pi E}(\delta_1 + \delta_2)\sigma. \tag{15}$$[2]

The discrepancy is

$$\frac{\Delta_2 L}{L} = \frac{\Delta L}{L} - \frac{\Delta_1 L}{L} = \frac{\delta_2 H^2}{8\pi E}. \tag{16}$$

This case differs from the previous one in that the longitudinal pull on the edges of the armatures is no longer transmitted to the cylinder, but is born by the armature itself. The force on the end of the tube can no longer be computed by means of the fictitious stresses but must be obtained by the separate calculation of the resultant z-components of the body-force and the surface force which act directly on the dielectric.

In what follows it will be assumed that the cylinder of dielectric extends beyond the edges of the armatures far enough so that the stray electric field at the end of the tube is of negligible intensity. Otherwise the longitudinal traction would depend on the length of the tube.

We choose the axis of the condenser as the z-axis and the plane A–A as the xy-plane. Denote the value of the electric force in the xy-plane by $\mathfrak{E}_0$.

The body force is easily computed. It is

$$P' = -\frac{(\delta_1 + \delta_2)}{16\pi}\int\int\int \frac{\partial(\mathfrak{E}^2)}{\partial z}\,dv \tag{17}$$

(Cf. equation (2).) The integral may be extended over the entire end of the tube although $\partial(\mathfrak{E}^2)/\partial z$ is sensibly equal to zero except near the ends of the armatures. Let L be the distance from the plane A–A to the end of the cylinder. Inserting the limits of integration we have

$$\begin{aligned} P' &= -\frac{(\delta_1 + \delta_2)}{16\pi}\int_0^{2\pi} d\theta \int_a^b r dr \int_0^L \frac{\partial(\mathfrak{E}^2)}{\partial z}\,dz \\ &= -\frac{(\delta_1 + \delta_2)}{8}\int_a^b r[\mathfrak{E}^2]_{z=0}^{z=L}\,dr \\ &= \frac{(\delta_1 + \delta_2)}{8}\int_a^b r\mathfrak{E}_0^2 dr. \end{aligned} \tag{18}$$

The surface force will now be computed. Quantities referring to the

[1] Sacerdote, J. de Phys. [3], XI., p. 196, 1901.

[2] Adams, l. c., equation 19.

inner and outer surfaces will be denoted by primes and double primes respectively. Let

$$\mathfrak{D}_z^2 + \mathfrak{D}_y^2 = \mathfrak{D}_r^2. \tag{19}$$

For a fluid

$$\delta_1 = \delta_2.$$

Consequently the expressions for the z-components of the forces per unit area on the inner and outer surfaces become

$$f_z' = \frac{\mathfrak{E}_z'\mathfrak{D}_r'}{2}\left[\frac{\delta_2 - \delta_1}{K}\right] \tag{20}[1]$$

and

$$f_z'' = -\frac{\mathfrak{E}_z''\mathfrak{D}_r''}{2}\left[\frac{\delta_2 - \delta_1}{K}\right].$$

Let $\mathfrak{E}_r'$ and $\mathfrak{E}_r''$ denote the radical components of the electric force in the tube of dielectric at its inner and outer surfaces respectively. Then

$$\begin{aligned} f_z' &= \left(\frac{\delta_2 - \delta_1}{8\pi}\right)\mathfrak{E}_z'\mathfrak{E}_r', \\ f_z'' &= -\left(\frac{\delta_2 - \delta_1}{8\pi}\right)\mathfrak{E}_z''\mathfrak{E}_r''. \end{aligned} \tag{21}$$

The total traction exerted on the surface is

$$\begin{aligned} P'' &= \int_0^{2\pi} d\theta \int_0^L af_z'dz + \int_0^{2\pi} d\theta \int_0^L bf_z''dz \\ &= \frac{\delta_2 - \delta_1}{8\pi}\int_0^L dz \int_0^{2\pi} d\theta[a\mathfrak{E}_z'\mathfrak{E}_r' - b\mathfrak{E}_z''\mathfrak{E}_r''] \\ &= -\frac{\delta_2 - \delta_1}{8\pi}\int_0^L dz \int_0^{2\pi} d\theta \int_a^b \frac{\partial(r\mathfrak{E}_z\mathfrak{E}_r)}{\partial r}dr. \end{aligned} \tag{22}$$

Both the curl and the divergence of the electric force vanish in the interior of the dielectric. Hence

$$\frac{\partial(r\mathfrak{E}_z\mathfrak{E}_r)}{\partial r} = r\mathfrak{E}_r\frac{\partial\mathfrak{E}_r}{\partial z} - r\mathfrak{E}_z\frac{\partial\mathfrak{E}_z}{\partial z} = \frac{r}{2}\frac{\partial}{\partial z}[\mathfrak{E}_r^2 - \mathfrak{E}_z^2]. \tag{23}$$

Therefore

$$P'' = \frac{\delta_1 - \delta_2}{16\pi}\int\int\int r\frac{\partial}{\partial z}(\mathfrak{E}_r^2 - \mathfrak{E}_z^2)\,drd\theta dz. \tag{24}$$

If we integrate with respect to z and θ, remembering that $(\mathfrak{E}_r^2 - \mathfrak{E}_z^2)$ vanishes at $z = L$ and reduces to $\mathfrak{E}_0^2$ at $z = 0$, we obtain

$$P'' = \frac{\delta_2 - \delta_1}{8}\int_a^b r\mathfrak{E}_0^2dr. \tag{25}$$

[1] Cf. equations (4).

The total longitudinal traction is

$$P = P' + P'' = \frac{\delta_2}{4}\int_a^b r\mathfrak{E}_0^2 dr, \tag{26}$$

or, if the variation of ϵ_0 with r be neglected

$$P = \frac{ad\delta_2 H^2}{4}. \tag{27}$$

The corresponding elongation is

$$\frac{\Delta' L}{L} = \frac{P}{2\pi adE} = \frac{\delta_2 H^2}{8\pi E}. \tag{28}$$

This is the discrepancy we have already found between the formulas of Adams and Sacerdote.

LIQUID ARMATURES.

In our discussion of condensers having adherent armatures we have tacitly assumed that these armatures are thin metal sheets. It has been a common practice on the part of experimenters in this field to use a conducting liquid for one of the armatures. That we can use the same formula for this case as for that in which both armatures are of metal foil is not immediately obvious. In fact we can show that if both armatures are liquid in contact with the dielectric cylinder the elongation is the same as if the armatures were nonadherent.

In the case of the typical adherent armatures first discussed the longitudinal traction is

$$P = \frac{K + \delta_2}{4}\int_a^b \mathfrak{E}_0^2 r dr.$$

The longitudinal pull exerted directly on the tube is

$$P_1 = \frac{\delta_2}{4}\int_a^b \mathfrak{E}_0^2 r dr.$$

Consequently the traction exerted on the edges of the armatures must be

$$P_2 = \frac{K}{4}\int_a^b \mathfrak{E}_0^2 r dr.$$

(This could also be proved directly.) Now in the case of liquid armatures this force is not transmitted to the tube but is borne by the liquid itself. Consequently we may neglect P_2 in computing the elongation of the tube. In this way we get the same formula as for the case of nonadherent armatures.

If the two armatures are of the same length we may assume that each carries half of the traction P_2. Then if one of them is a conducting liquid we may compute the elongation from (9) if we substitute $K/2$ for K.

However, the usual experimental arrangement is to have the liquid armature very much longer than the coating of metal foil. With this arrangement the two armatures do not carry equal longitudinal tractions. In fact there is no longitudinal pull on the liquid dielectric at all, for the lines of force arrange themselves as indicated in Fig. 2, and the metal

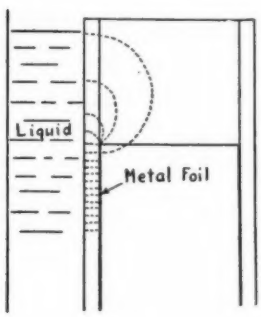

Fig. 2.

foil must carry the whole of the traction P_2. Consequently the whole stress is transmitted to the dielectric and the elongation is the same as if both armatures were metal sheets.

THE EXPERIMENTS OF MORE AND OF SHEARER.

The above work shows that if the general theory of electrostriction is correct the experimental results of More and of Shearer must be reconciled with the Sacerdote formula, (9). Moreover, it shows that the principal term of (9), $(KL/8\pi E)(V/d)^2$, is not due to a direct action of the electric field on the dielectric but to the longitudinal pull on the edges of the armatures. If there were no direct electrostrictive action (as has been suggested) we would still expect an elongation given by the formula

$$\frac{\Delta L}{L} = \frac{K}{8\pi E}\left(\frac{V}{d}\right)^2. \tag{29}$$

Since the publication of the work of More and of Shearer, Professor Adams[1] has computed the value of the combination of constants $[\delta_1 - (\delta_1 + \delta_2)\sigma]$ for several different kinds of glass from the experiments of Wüllner and Wien on the change in the capacity of cylindrical glass condensers when they are stretched. The computed values vary from $+0.68$ to -2.85. For glasses for which this combination of constants has a negative value the formula (9) predicts a smaller elongation than does (29). Hence the experimental results of More and of Shearer

[1] E. P. Adams, l. c., p. 895.

are more easily explained if the general theory of electrostriction is assumed to be correct than otherwise.

The elongation predicted for tube No. 5 of Professor More's second series of experiments[1] by (9) is 3.6×10^{-6} cm. or 6 divisions of Professor More's micrometer scale if the extreme value -2.85 of the combination of constants $[\delta_2 + (\delta_1 + \delta_2)\sigma]$ is used. The smallest elongations observed for this tube were 5 and 7 micrometer divisions. Possibly an appreciable fraction of these minimum observed elongations was due to heat but the discrepancy between theory and experiment can hardly be considered to amount to a contradiction.

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[1] L. T. More, Phil. Mag. (6), 6, p. 13, 1903.

RETROGRADE RAYS FROM THE COLD CATHODE.

By Orrin H. Smith.

J. J. THOMSON,[1] as early as 1897, showed that a system of rays of an entirely different character from the cathode rays accompanies the cathode beam. He found that these rays proceed normally from the face of the cathode, that they are not appreciably deflected by a permanent magnet, and that they possess very little, if any, power of producing phosphorescence.

In 1906 Villard[2] gave an account before the French Academy of the rays accompanying the cathode beam which are not so readily deflected as the cathode beam but which were deflected in such a direction and by such an amount as would be expected of the "kanal strahlen." He noticed that in a mixture of oxygen and hydrogen (or water vapor) the cathode rays produced a luminescence characteristic of oxygen, but when these were deflected aside by a magnet there remained rays which produced a luminescence characteristic of hydrogen. He explained their presence by saying that they were the positive canal rays which fall against the cathode and rebound. To explain their rebounding beyond the limits of the cathode dark space, he assumed that the potential fall underwent rapid variations or was even discontinuous. A stroboscopic test showed this to be true; however, this was to be expected since he used a transformer to produce the discharge.

The following year Thomson[3] showed, independently, that these rays were deflected by strong electric and magnetic fields and that they possessed considerable mass. In a later work he observed that they were very feeble under the most favorable conditions of vacuum, discharge potential, etc., and were exceedingly feeble when the gas pressure in the discharge tube was very low. In this latter respect they were quite different from canal or positive rays. Employing a tube having an opening of about .5 mm. in diameter he obtained a photograph which showed that these rays contain (*a*) positively electrified atoms and molecules of hydrogen, (*b*) positively electrified atoms of oxygen, and (*c*) negatively electrified atoms of hydrogen and oxygen. The photograph

[1] Proc. Camb. Phil. Soc., IX., p. 243, 1897.
[2] Comptes Rendus, CXLIII., p. 673, 1906.
[3] Phil. Mag., XIV., p. 359, 1907.

showed the intensity of the lines corresponding to the negative ions to be greater than that of the positive ions. With the ordinary positive rays the positive lines are the more intense.

The conditions under which retrograde rays are produced are quite different from those that obtain for the ordinary positive rays and for this reason it seemed worth while to repeat and extend Thomson's investigations.

Thomson does not find the molecule of oxygen with the negative charge while in this investigation the molecule of oxygen and the molecule of hydrogen are the only carriers obtained with a negative charge, no atoms appearing at all. The presence of helium in the discharge chamber apparently makes no difference in the photographic result.

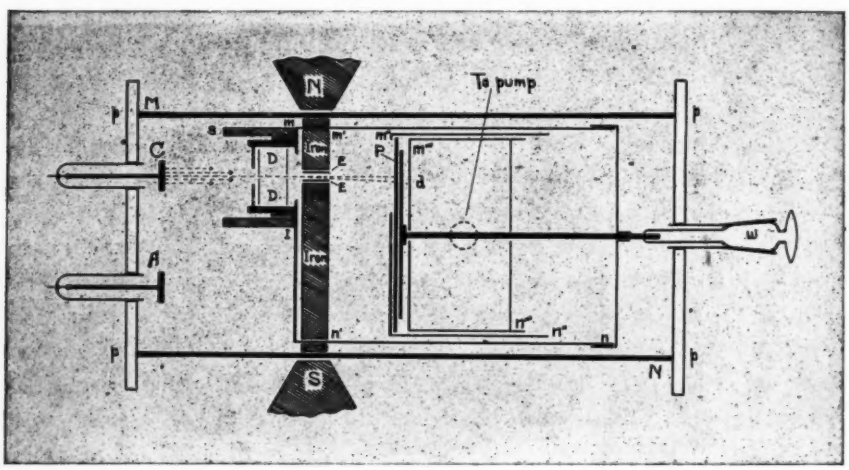

Fig. 1.

Top View. *MN*, containing vessel of glass; *pp*, glass end plates; *mn*, large brass cylinder; *m'n'*, magnetic field extensions; *EE*, electrostatic field plates, connections to which are not shown; *m''n''*, plateholder; *P*, photographic plate mounted on disc *d*, supported by telescoping cap *m'''n'''*, and turned by winch *w*; *DD*, aluminum diaphragms; and *SI*, iron shield.

It appears that Thomson was unable to use a tube of less than .5 mm. bore, while in this investigation traces were obtained with a tube and set of diaphragms having openings of about .05 mm. thus producing sharp lines on the plate which made possible more accurate measurements.

Owing to the short range at which these rays were obtained on the photographic plate, the increased sharpness of the lines, and the restricted range of their velocities due to a restricted cathode dark space, it was possible to obtain some evidence on the question as to whether the power of a particle to affect a photographic plate is a function of its

velocity, momentum, or kinetic energy. This evidence seems to indicate that it is a function of the kinetic energy and that the mean value is about 7.4×10^{-9} ergs.

The apparatus, shown in Fig. 1, and the manipulation is essentially the same as that described by Knipp[1] except that a cold cathode was used instead of the Wehnelt cathode and the discharge was produced by an induction coil. The cathode was just like the anode and similarly placed facing the line of the tube and the diaphragms.

A large Leeds induction coil was operated on ten storage cells. The vacuum was maintained with the aid of a large charcoal bulb dipping into liquid air. In general the vacuum improved with sparking. After a few runs it was found that the liquid air could be removed after about ten minutes from starting, and, as the sparking and pumping continued, it could be dispensed with altogether. Finally the vacuum was so easily maintained that it was necessary to keep the pump itself turned off for about three fourths of the time.

There is a point of interest in connection with the charcoal bulb. It was left on the apparatus for weeks after its use was found unnecessary, remaining all the while at the nearly constant room temperature. The pumps were unable to produce a vacuum of .005 mm. in fully three hours' time when starting from atmospheric pressure, and this was the case whether the bulb was heated for an hour during that time or not. However, if it was pumped to a pressure of one or two mm. and left to stand for ten to fifteen hours then upon starting the pumps a vacuum of .005 mm. could be attained in twenty to thirty minutes. This, strangely, was true even when the vacuum had been let down for a very few minutes and then the pumps started again immediately.

The photographic plate used was Seed's Yellow Label lantern slide plate. This plate is very slow and hence produces great contrast which is the thing desired. Thomson[2] points out that the large ions affect only the surface of the film and do not penetrate like the faster moving electrons, into the film. Hence the plate best suited for this work is one that is slow and that has a thin film with a high percentage of silver. The best traces that could be gotten in this investigation were in many instances so thin that they could hardly be seen. They were obscured easily by the slightest fogging. For this reason a fast plate could not be used. Seed's Gilt Edge Number Twenty-seven plate was tried but in every instance fogging obscured the lines. The Double Coated Cramer Crown plate was tried and found to be entirely too sensitive. Some

[1] PHYS. REV., XXXIV., p. 215, March, 1912.

[2] Thomson, Rays of Positive Electricity, p. 4.

experience by another member of the department obviated the necessity of trying the Cramer X-ray plate. As an instance to show that these carriers affect only the surface of the film, the author gently stroked the film under water with a fine camel's hair brush to remove foreign particles and it was found that, in some cases, the lines were entirely obliterated. Further, after the negative had dried the lines could be obliterated by breathing on the film and wiping it gently with a soft cloth. In both cases, other than erasing the lines, no further change could be detected in the film. It was found advantageous to put some alum in the fixing bath to harden the film. The developer used was ordinary hydrochinon, the time of development being from six to twelve minutes.

The time of exposure varied from thirty minutes for the small to three hours for the larger deflections. There seems to be a limit to the intensity that is obtainable, for after a certain length of exposure the intensity of the lines did not apparently increase with further exposure. This was true for long or short development or even when they were exceedingly dim. This is in agreement, however, with the theory that they affect only the surface of the film.

Thomson found that the retrograde rays were best obtained when the gas pressure was not too low. The present photographs bear out that fact very well. If the vacuum was kept about .002 to .004 mm. scarcely any trace of the rays could be found on the plate. The best pressure for their production seems, from this investigation, to be between .015 and .008 mm.

There is always a central spot that is undeflected which is probably due to neutral carriers that were negative originally but which lost one electron before they got into the deflecting fields. It would seem from this that a moving particle need not be charged in order to affect a photographic plate. It is quite evident that the velocity of an uncharged particle must be above a certain value otherwise a plate would be affected by exposure to the air in a dark room due to no other agency than to the velocity of the air molecules produced by ordinary heat agitation. The mean of this velocity at 0° C. for the hydrogen molecule is about 2×10^5 cm./sec. and for the oxygen molecule about 4.5×10^4 cm./sec. Whether the ability of a moving particle to affect a photographic plate is due to its momentum or its kinetic energy, or simply to its velocity, is not definitely known. It seems reasonable to expect, however, that it should be a function of one of these. On a number of the plates the lines were distinct enough to locate approximately the place where the slowest ions would strike, *i. e.*, those that had just sufficient velocity to affect the plate. These points were found, in every

case, to be well within the limits of the field, *i. e.*, so far as the limits of the apparatus are concerned the lines might have extended farther from the origin. It occurred to the author then to assume that there were particles which struck beyond the last points of the visible trace but whose velocity was not sufficient to cause them to affect the film. If the coördinates of the last visible point in each line be measured and v and e/m determined, then, for all such points, we should get a constant, showing whether this minimum effect on the plate is a function of the velocity, the momentum, or of the kinetic energy of the moving ion. Table I. shows values which are proportional to the velocity, momentum, and kinetic energy for the points in question on sixteen different lines. It can be seen that the values for the kinetic energy are nearly constant while the values for the velocity and the momentum are not constant. It thus appears that the power of a particle to affect a photographic film probably depends on its kinetic energy. The mean of these values of the kinetic energy is, from Table I., 7.4×10^{-9} ergs which is the minimum required. This value would probably be different for an electron because of its size. It is somewhat larger than the energy required to produce an ion which is 1.63×10^{-11} ergs. The above value (7.4×10^{-9}) was calculated from data obtained from this investigation, except for the value of e, by the formula

$$\text{kinetic energy} = 1/2 \cdot m/e \cdot e \cdot v^2.$$

The value of e was taken as 1.55×10^{-20}.

TABLE I.

Photographic Plate.	Line.	Constant × Velocity.	Constant × Momentum.	Constant × Kinetic Energy.
75	Upper	6.04	18.36	111.0
76	Upper	6.54	17.99	117.6
76	Lower	1.36	87.18	118.6
85	Upper	6.37	12.50	79.0
85	Lower	1.52	52.50	79.7
86	Upper	6.23	12.71	79.2
86	Lower	1.53	52.48	80.4
87	Upper	7.27	19.40	102.5
87	Lower	1.69	60.50	102.3
88	Lower	1.37	61.70	84.5
94	Upper	5.55	10.71	106.4
94	Lower	1.77	36.28	100.4
95	Upper	7.41	15.03	91.1
95	Lower	1.64	51.50	84.47
96	Upper	6.35	17.70	112.5
96	Lower	1.42	59.36	84.26

It can be seen from Table I. that, even though the values of the kinetic energy vary somewhat, the values for a given plate as a rule are more nearly alike. Plate ninety-six furnishes the greatest variation from this rule. It might be reasonable to expect that different emulsion numbers would reveal slightly different kinetic energies required to affect the film. Several emulsion numbers are represented in these data.

The photographs taken with the apparatus in the last refinement, while clear and capable of accurate measurement, do not lend themselves to reproduction and hence are omitted. The important dimensions are as follows:

Length of electrostatic field	1.10 cm.
Length of magnetic field	1.10 cm.
Distance from point of emergence to plate	1.58 cm.
Length of triangular test coil[1]	2.52 cm.
Base	.63 cm.
Number of turns	19.

The negative lines show distinctly the parabolic heads which are not in evidence on the positive lines. It was evident from nearly all the plates exposed that the negative carriers are in preponderance over the positive ones. This seems reasonable to expect since the distance to the plate is, for the lower pressures, within the limits of the mean free path and it is necessary to assume that every positive carrier has lost two electrons between the outer limits of the dark space and the deflecting fields. If this is true we should expect that the lines due to the positive carriers would not be as sharp as those due to the negative carriers, the ions being deflected somewhat from their true path in the process of losing an electron. Most of the photographs bear this out. It is somewhat surprising, in consideration of the foregoing, that this preponderance is not greater than the photographs seem to indicate unless the negative ion is more unstable than the positive ion. An additional suggestion in the same line comes from a study of Thomson's photographs of positive rays, in a great many of which the negative counterpart is very weak or cannot be seen at all on the prints when the positive lines are very pronounced. The positive lines do not have the distinct parabolic head that the negative lines have. They are also broader and more diffuse. Joining the parabolic head to the center is a line due to the secondary rays of Thomson. This is shown particularly in one exposure where the electric field overlapped the magnetic so that the secondary line does not join straight on to the head of the parabola. The data for exposure number eighty-five, are given in Table II. This

[1] Thomson, Rays of Positive Electricity, p. 10.

indicates that the carriers which produced the two lines are the molecules of hydrogen and oxygen respectively. The measurements of the coördinates were made with an ordinator composed of a frame to which the plates could be fastened so that there was a movable point above the plate capable of being carried in either of two directions perpendicular to each other by micrometer screws. A Grassot fluxmeter was used to determine the strength of the magnetic field.

TABLE II.

Photographic Plate Number 85.

Measurements for the Upper Line.

Position.	x in mm.	y in mm.	$v \times 10^{-7}$ cm./sec.	$e/m \times 10^{-4}$	Electric Atomic Weight.	Carrier.
1	3.34	6.42	6.37	.509	1.97	H_2
2	2.46	5.48	7.39	.504	1.99	"
3	1.88	4.77	8.96	.500	2.00	"
4	1.12	3.51	10.38	.458	2.18	"

Measurements for the Lower Line.

1	3.34	1.53	1.52	.029	34.5	O_2
2	2.46	1.31	1.77	.0288	34.8	"
3	1.88	1.16	2.04	.0296	33.8	"
4	1.12	.84	2.50	.0260	38.5	"

Time of exposure, 3.25 hours.
Gas pressure varied between .008 and .018 mm.
Electric deflecting field, 965 volts.
Magnet current, 4.25 amperes.
$A = 8{,}040$, $B = 267 \times 10^9$

All the photographs were exposed with residual air in the discharge chamber except number 88. In this instance it contained some helium but no traces appear in the photograph, in fact in no case does anything appear in any of the photographs except the lines due to the molecules of hydrogen and oxygen. In some cases the positive rays are not visible.

The data show very well how the velocity varies for the carriers striking at the various points along the parabola, that it decreases with increase of distance from the undeflected spot. The value of v and e/m obtained for the smaller values of the electric field are in general less reliable than for those for which the deflection is larger. The "electric atomic weight" of a carrier Thomson[1] has defined as the ratio of m/e for that carrier to m/e for the atom of hydrogen.

[1] Phil. Mag., XXI., p. 234, Feb., 1911.

It was noticed in connection with these experiments that the discharge in the chamber passed more easily with the presence of a transverse magnetic field. Earhart[1] has shown that this is true for a longitudinal field.

SUMMARY OF CONCLUSIONS.

The results of this investigation may be summarized briefly as follows:

1. When obtaining retrograde rays in residual air the molecule of hydrogen appears on every plate accompanied by a heavier carrier which in most cases is the molecule of oxygen. The velocities obtained by the author are smaller than those obtained by Thomson. This is due to the position of the cathode with reference to the small canal through which the carriers pass, the dark space extending beyond the near end of this tube and hence the carriers not attaining their maximum velocity.

2. The negative lines are clearer and sharper than the positive; probably because of the disturbance to the path of the positive particles in the process of becoming positive.

3. Retrograde rays can be obtained with a canal having a bore of about .05 mm. diameter. The best range of pressures for their production is between .008 and .015 mm. of mercury.

4. The power of a moving particle to affect a photographic plate seems to be a function of its kinetic energy. The minimum required for the heavy carriers is of the order 7.4×10^{-9} ergs, which is larger than the energy required to produce an ion, however, there is evidence in favor of the view that this value may depend somewhat on the emulsion on the plate.

In conclusion I wish to express my thanks to Professor A. P. Carman for the excellent facilities placed at my disposal and to Dr. C. T. Knipp for his interest and help in carrying on the investigation.

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[1] PHYS. REV., Feb., 1914.

ON THE "CURRENT-DEFLECTION" METHOD FOR DETERMINING BALLISTIC CONSTANTS OF MOVING COIL GALVANOMETERS, WITH A NOTE ON THE NON-UNIFORMITY OF MAGNETIC FIELDS IN SUCH INSTRUMENTS.

By Paul E. Klopsteg.

I. Theoretical.

(*a*) *Statement of Formula for Method.*—The determination of the quantities involved in the equation[1]

$$K = \frac{T}{2\sqrt{\pi^2 + \Lambda^2}} k\rho^{\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}}, \qquad (1)$$

which reduces to

$$K = \frac{T}{2\pi} k\sqrt{\rho},$$

when Λ is small in comparison with π, is easily accomplished; and simple apparatus, found in any laboratory, is used. In these equations K represents the ballistic constant,[2] ρ is the ratio of one amplitude of the coil to the one next following, T is the complete period of the coil and Λ its logarithmic decrement; k is the current constant,[3] i/d, of the instrument, where, in the ordinary method of determination, d is the deflection from the null position caused by the current i.

(*b*) *Objection to the Use of k as Ordinarily Determined, and Proposal of Modified Method.*—It seems that equation (1) is not commonly used, except for instructional purposes in connection with the theory of the ballistic galvanometer, the presumable reason being that the values which it gives for the constant are not often in agreement with those given by the direct methods, the differences ranging from a fraction of one per cent. to as much as three or four per cent. The major part of the error when damping is small occurs in the value of k; with strong damping, large errors are as likely to occur in Λ as in k. The formulæ presuppose ideal conditions as to uniformity of field and symmetry of

[1] Kohlrausch, Lehrbuch der praktischen Physik, 12th edition, 1914, p. 525.

[2] The open coil type of instrument with a circular scale at 50 cm. from the mirror is here being considered; K in coulombs per centimeter.

[3] In amperes per centimeter.

position of the coil with respect to the magnet; these conditions are far from being realized in most galvanometers, as has been previously indicated by the writer,[1] and as will be shown at greater length in this paper. Since, ordinarily, the field is by no means uniform, the value of k is not truly constant, but depends upon the angular position of the coil.

The value of k to be used in formula (1) is the value which depends upon the average field intensity *at the null position of the coil*, which, of course, is the position occupied by the coil at the instant the greater part of the discharge passes through it. This fact suggests that it might lead to greater accuracy if, instead of producing a given deflection by means of a current, one were to produce a deflection by turning the torsion head from which the coil is suspended through a certain angle, and then to determine the current necessary to bring the coil back to its null position. The coil would then evidently be in the very position in the field which it occupies at the moment of the ballistic impulse. The question at once arises whether the current constant should, with this manipulation, have the same value as that which would be obtained by the ordinary method if the field were truly uniform. For it is evident that in the proposed procedure the spiral lower suspension is, when the coil has been returned to its null position, in the same condition of strain as it was before the coil was deflected by turning the torsion head.

To investigate this point, we may proceed as follows:

Let i_1 = current through coil equivalent to torque per unit angle of the upper suspension; and

i_2 = current equivalent to torque per unit angle of the lower suspension;

also let the upper torsion head be turned through an angle d_0 (in terms of centimeters on the circular scale), producing an angular deflection d of the coil. Then

$$i_2 d = i_1(d_0 - d),$$

from which

$$d = i_1 d_0/(i_1 + i_2).$$

Now let a current i be sent through the coil, just sufficient to bring it back to the null position; since the lower suspension is now free from strain, $i = i_1 d_0$. Consequently

$$k = i_1 + i_2.$$

The same result is obtained when the assumption is made that the lower suspension is given sufficient twist to produce the deflection d of the coil. By twisting either suspension, therefore, the restoring couple of the

[1] PHYSICAL REVIEW, N.S., 5, 266, 1915.

system is unchanged. This leads to the important conclusion in connection with the usual condition[1] of the suspensions in galvanometers, that within limits, no matter what the conditions of strain of the suspensions at the null position, the elastic constant of the suspensions is the same as though the suspensions were free from strain.[2]

(*c*) *Precautions to be Observed in Determining k by the Method Here Outlined.*—In determining the current constant by the modified method, one source of error should be kept in mind, namely, that in most instruments—unless special attention is paid to this point—the prolonged axis of the coil does not pass through the point of attachment of the upper suspension. This may, for example, be due to a bent terminal, or to eccentricity of the torsion head. When the torsion head has been turned through a certain angle and the coil brought back to zero deflection by means of a suitable current, the position of the coil is not usually identical with the "original" null position,[3] but may be shifted into a region of the field at which the intensity differs appreciably from that at the original null.[4] The current constant will then be affected by an error proportional to the difference between the field intensities at the two positions. The error from this cause, for a given deflection of the coil, is less with an upper suspension having large torque per unit angle than in one with small torque; for in the latter case the upper torsion head must be turned through a larger angle to produce a given deflection of the coil than in the former. The gain in accuracy, however, is offset by the smaller sensitivity with a stiff upper suspension. It should therefore be observed, especially when a galvanometer with weak control is being used, that the upper suspension be as nearly coincident with the vertical axis of symmetry of the instrument,[5] and the coil as near the ideal position, as careful levelling can bring them.

Another precaution to be observed is that of using the proper value of the logarithmic decrement corresponding to a given throw, especially when there is considerable damping; since the logarithmic decrement is not constant, its value must be determined for any given amplitude. A method for doing this is described in the experimental section of this paper.

[1] This condition being that, at the null position of the coil, one of the suspensions is twisted, necessitating a twist of the other suspension in the opposite direction for equilibrium.

[2] It has been shown by Pealing (Phil. Mag., 29, 203, 1915) that "bifilar effect" of a phosphor bronze ribbon is negligible in its effect upon the elastic constant for torque.

[3] Meaning the position which the coil occupied before it was deflected by means of the torsion head.

[4] Klopsteg, loc. cit.

[5] Loc. cit

II. EXPERIMENTAL.

A. Tests on Uniformity of Magnetic Field.

(*a*) *By Means of Current Sensitivity.*—A 3-mil phosphor bronze strip upper suspension was selected with terminal wires such as are provided by the manufacturers. The upper terminal wire was bent so as to displace the suspension fiber about 2 mm. from the vertical axis of the instrument. If the plane of the coil, by suitable means, is kept parallel to its original position while the upper torsion head is turned through 360 degrees, each point of the coil describes a circle the radius of which is the amount of the eccentricity. To accomplish this end, the upper torsion head was turned through a small angle and sufficient current was sent through the coil to return it to the null reading. The angle and current were accurately measured. The process was then repeated sufficiently often to make a complete revolution of the torsion head. The current per degree, for different positions of the coil in the field, was found to vary almost 10 per cent. When the suspension terminals are very short, so as to eliminate the eccentricity mentioned in the first case, the current per degree is found constant over the whole range within which the instrument is used, showing that no matter what the twist in the fiber, the coil was, in each measurement, in the same position relative to the field. Similar results were obtained with a 1.5-mil phosphor bronze strip suspension.

(*b*) *By Means of Ballistic Sensitivity.*—The results of the preceding experiment make it seem probable that the differences which one frequently observes in the ballistic throws of an instrument with equal quantities of electricity but in opposite directions from the same null point[1] may be ascribed to the non-uniformity of the galvanometer field. To investigate this point, an experiment suggests itself, as follows: Using an upper suspension fiber with short terminals, so as to avoid eccentricity, one may, by means of the upper torsion head, shift the coil into various angular positions with respect to its usual null position; for each particular position one may take throws in opposite directions with equal quantities.

This was carried out, starting with a shift of about 10 degrees towards the "red" side of the scale, and, in small steps, working over to the "black" side. The results[2] are shown graphically in Fig. 1. The instrument was moderately damped, and the same quantity of electricity was used for every throw. Abscissas are given in degrees of displacement of

[1] Peirce, B. O., Am. Acad. Proc., 42, 161, 1906.

[2] The results here given are from a single instrument; but they may be observed, to a greater or smaller degree, in any instrument of the type described.

the various positions from the ordinary null, ordinates represent centimeters of throw from each position. "*B*" refers to the "black" side of the scale, "*R*" to the "red" side. The two curves in the figure are seen to cross at a point about 0.8 degree from the null toward the "red," indicating that if one were to shift the coil into this position as a new null, equal throws would be obtained *with this particular quantity*. With larger or smaller quantities the curves would not necessarily intersect at the same abscissa.

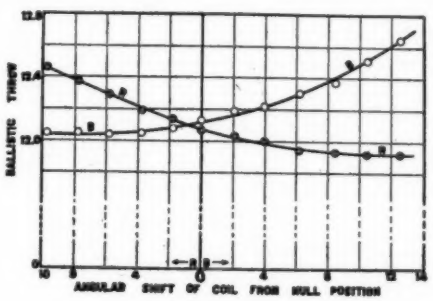

Fig. 1.

Clearly, with a given quantity, the magnitude of the throw depends upon two factors: the intensity of the field at the null position of the coil, and the amount of damping in the region through which the coil swings. The former is the same for any given position of the coil, regardless of the direction of swing; the latter depends upon the nature of the magnetic field. The experiment shows non-uniformity in two ways: first, differences in throws in the same direction from different null positions; and second, differences between throws in opposite directions from the same null.

B. Method of Finding Logarithmic Decrement to be Applied to a Throw of Given Magnitude.

When the damping is slight, it is quite sufficient to find the logarithmic decrement or the damping factor in the ordinary manner, using two successive elongations somewhere near the value of the throw to be corrected, remembering that these elongations are to be taken on the same side of the null on account of the probable difference in the damping on the two sides. For highly damped throws—the motion of the coil still being periodic—the above method may not give the value with sufficient accuracy because of the great difference between two successive elongations on the same side of the null. The procedure in this case is as follows: The resistance in the circuit having been adjusted to the same value as that which is to be used in the measurements, the coil is started swinging from a large displacement. As many sets of readings of successive elongations on the same side of the null are taken as is considered necessary for the desired degree of accuracy. One of these sets is first plotted, using the elongations as ordinates, and the time, expressed in terms of the complete period of the coil, as abscissas. The other sets may then

be "fitted in," using the same time unit, and the curve drawn. To apply the curve to the determination of the logarithmic decrement corresponding to any particular amplitude, we find the intersection of the curve with the horizontal line at the amplitude in question; from this point we pass to the left a distance equivalent to a half time-unit, and find the amplitude corresponding to this instant. The natural logarithm of the ratio of the amplitude so found to the amplitude chosen is the logarithmic decrement sought. It is well to remember that this correction is slightly affected by the zero shift due to magnetic impurities;[1] for in an actual measurement one eliminates the zero shift as suggested by Zeleny, while the curve here described is obtained from readings of the positions of the coil as it executes complete swings. The amount of the error has not been determined, but in the ordinary instrument its effect upon the results of measurements is certainly less than 0.2 per cent.

C. Test of Modified Method for Obtaining K.

The results shown in Tables I. and II. were obtained in a series of determinations, the purpose of which was not to approach the limit of accuracy attainable by this method of finding the ballistic constant, but rather to determine the reliability of the method under ordinary laboratory conditions. Each quantity used in the formula was determined from but a single careful observation. The instruments were levelled, but it was not attempted to eliminate absolutely the possible eccentricity due to bent terminals. The galvanometers are the ones which are constantly in use in the undergraduate laboratory; no tests were made to see if the scales were accurately circular, or if they were exactly 50 cm. from the mirrors.

A comparison of the results shows that even without these precautions the modified method is superior to that commonly used. In the writer's opinion one is justified, when careful work is done and several determinations of each quantity are made, in assuming an accuracy of 0.2 per cent. in the results.

It would greatly facilitate the manipulation of returning the coil to its normal position from the deflection produced by turning the upper torsion head if the latter were provided with a tangent screw, permitting accurate adjustment at the first trial.

III. SUMMARY.

(*a*) A method is proposed for obtaining the current constant which is used in the equation for obtaining the ballistic constants of moving coil galvanometers, and possible sources of error are discussed.

[1] A. Zeleny, PHYS. REV., O. S., 23, 400, 1906; PHYS. REV., O. S., 32, 297, 1911.

TABLE I.

Galvanometers provided with 3-mil phosphor bronze ribbon upper suspensions; damping sufficiently small to admit of use of approximate form of eq. (1).

Galv. No.	K, Coulombs/Cm. $\times 10^7$.		
	Usual Method.	Modified Method.	Condenser[1] Method.
I	2.901	2.880	2.873
III	3.440	3.469	3.464
VI	2.957	2.931	2.938
VII	3.161	3.162	3.157
IX	5.314	5.326	5.324
XI	2.804	2.793	2.793
XIII	2.961	2.961	2.960
XV	3.009	2.993	2.994
XVI	2.814	2.799	2.802

TABLE II.

Galvanometers provided with 1.5-mil phosphor bronze ribbon upper suspensions; small damping.

Galv. No.	K, Coulombs/Cm. $\times 10^7$.		
	Usual Method.	Modified Method.	Condenser[1] Method.
II	1.561	1.561	1.563
IV	1.921	1.932	1.931
VIII	1.479	1.459	1.454
X	1.989	1.970	1.966
XII	1.650	1.669	1.665

(*b*) Experiments are described in which these sources of error are investigated, leading to suggestions for minimizing these errors. Principally the errors are due to non-uniformity of the magnetic field and to insufficient levelling of the instrument.

(*c*) Results of tests of the proposed method are given which indicate that with care the method is reliable to 0.2 per cent. accuracy.

(*d*) The suggestion is made that galvanometers to which the method is to be applied be provided with a tangent screw for fine adjustments of the torsion head.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
January 12, 1916.

[1] Condenser with Bureau of Standards certificate, charged by means of cadmium standard cells, using free charge only. See A. Zeleny, PHYS. REV., O. S., 22, 1906, p. 65; O. S., 23, 1906, p. 409.

THE CORRECTION FOR THERMOELECTRIC CURRENT TO BE APPLIED TO THE THROW OF A BALLISTIC MOVING-COIL GALVANOMETER.

BY PAUL E. KLOPSTEG.

THEORETICAL.

ON account of the simplicity and convenience of the condenser method of determining the ballistic constant of a moving-coil galvanometer, one is likely to use it in preference to the more complicated inductance-coil methods in making an accurate determination of the constant. This is especially true when the instrument is used on open circuit, in which case there are no particular difficulties to be overcome in making a precise measurement. When the discharge to be measured takes place in the closed galvanometer circuit,[1] the constant should be determined under similar conditions, because of the dependence of the ballistic throw upon damping. When using a standard condenser for obtaining the constant on closed circuit it is necessary, immediately after the condenser has been discharged, to connect the galvanometer through a resistance such that the total resistance is the same as it is to be in the circuit with which the measurements are to be made. This is easily accomplished by means of special keys or switches.

Upon changing from open to closed circuit one is likely to encounter a thermoelectric current[2] which deflects the coil either in the same direction as or opposite to that of the throw caused by the impulsive discharge. The time of reaching the maximum elongation from the null point is not the same in the case of a current of very short duration as in the case of a deflection due to a steadily impressed electromotive force. Were these time intervals alike, the correction could be made by algebraic addition of the maxima. The relation between them depends upon the amount of damping in the circuit, and is therefore different for different values of the logarithmic decrement. Zeleny[3] has given an experimental curve for correcting the observed throw and obtaining from it

[1] Zeleny and Erikson, Manual of Physical Measurements, 3d ed., p. 172; Smith, Electrical Measurements, p. 195.

[2] A. Zeleny, PHYS. REV., O.S., 23, 1906, 414. Another method of determining the constant, not yet published, obviates the necessity for making the correction for thermoelectric effect.

[3] Loc. cit.

the magnitude of the throw which would have resulted had there been no thermo-electromotive force in the circuit.

It is the purpose of this paper to obtain a relation between the quantities involved, by means of which the correction may be made. The following notation will be used:

I_0 = moment of inertia of the coil;
$2f$ = proportionality constant between damping moment and angular velocity of the coil;
$M = nAH$, where n is the number of turns and A the mean area of the coil, and H the field intensity;
q^2 = elastic torque constant of the suspensions;
e = impressed electromotive force;
r = total resistance in the circuit;
i = current at instant t;
θ = angle of deflection;
Λ = logarithmic decrement of coil;
ρ = ratio between successive elongations;
T = complete period of coil.

When a steady electromotive force exists in the galvanometer circuit, the resulting motion of the coil may be expressed by the differential equation of moments

$$I_0 \frac{d^2\theta}{dt^2} + 2f \frac{d\theta}{dt} + q^2\theta = Mi, \tag{1}$$

one solution of which is

$$\theta = \epsilon^{-at}(A \cos bt + B \sin bt) + \frac{Mi}{q^2}, \tag{2}$$

the condition for this solution being $I_0q^2 > f^2$. A and B are the constants of integration, $a = f/I_0$ and $b = \sqrt{(q^2/I_0) - (f^2/I_0^2)}$. Imposing the initial conditions, $t = 0$, $\theta = 0$, $d\theta/dt = 0$, we find

$$\theta = \frac{Mi}{q^2}\left[1 - \epsilon^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right)\right]. \tag{3}$$

This equation represents a damped motion of period $T = 2\pi/b = 2\Lambda/a$, θ having the value Mi/q^2 after a sufficiently long time has elapsed. The first maximum elongation is reached at the instant

$$t = \frac{\pi}{b} = \frac{T}{2}, \tag{4}$$

found by equating $d\theta/dt$ obtained from (3) to zero and solving for t.

Putting this value of t in (3) and solving for θ_d', the first maximum elongation,

$$\theta_d' = \frac{Mi}{q^2}(1 + \epsilon^{-\Lambda}). \tag{5}$$

The equation of motion of the coil when there is no torque tending to produce a definite displacement of the coil from its null position is

$$I_0 \frac{d^2\theta}{dt^2} + 2f \frac{d\theta}{dt} + q^2\theta = 0, \tag{6}$$

the solution of which, for the condition $I_0q^2 > f^2$, is the same as eq. (2) with the exception of the last term, which in this case is zero. Using the initial conditions $t = 0$, $\theta = 0$, $d\theta/dt = MQ/I_0$ to evaluate the constants of integration, the result is

$$\theta = \frac{MQ}{I_0} \epsilon^{-at} \left(\frac{1}{b} \sin bt\right). \tag{7}$$

This is also a damped vibratory motion of the same period as that given by eq. (3), provided a and b have the same values, respectively. Its first maximum elongation occurs at the instant

$$t = \frac{1}{b} \tan^{-1} \frac{b}{a}. \tag{8}$$

Suppose, now, that a quantity of electricity Q in a condenser is discharged through the galvanometer and that the circuit is immediately closed; and that, upon closing, a thermoelectric current flows. If the direction of the steady current is the same as that of the discharge, the resulting maximum is greater than it would be in the absence of the thermo-electromotive force; if the direction of the current is opposite to that of the discharge, the resulting throw is smaller. Clearly, the correction to be applied to the observed throw is the angle attained in a steady deflection[1] during the time interval required for a ballistic maximum, obtainable with the aid of equations (8) and (3). Designating this angle by ϕ, we find

$$\phi = \frac{Mi}{q^2}\left[1 - \epsilon^{-\frac{a}{b}\tan^{-1}\frac{b}{a}}\left(\cos \tan^{-1}\frac{b}{a} + \frac{a}{b} \sin \tan^{-1}\frac{b}{a}\right)\right],$$

or, in terms of the logarithmic decrement,

$$\phi = \frac{Mi}{q^2}\left[1 - \rho^{-\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}} \cdot \frac{2 \cdot \frac{\Lambda}{\pi}}{\sqrt{1 + \frac{\Lambda^2}{\pi^2}}}\right]. \tag{9}$$

[1] Assuming steady deflection small in comparison with throw. Zeleny mentions 1/3 as he maximum permissible ratio. For further discussion see below.

In order to make this equation for the correction applicable to any galvanometer of the type under discussion, it is convenient to express ϕ in terms of either of two ratios: $R_1 = \phi/d$, giving its value as a fraction of the steady deflection; or, $R_2 = \phi/\theta_d'$, which gives the value as a fraction of the *first maximum due to the steady current.* The value of d, from eq. (3) (putting $t = \infty$) is seen to be Mi/q^2; consequently,

$$R_1 = 1 - \rho^{-\frac{1}{\pi}\tan^{-1}\frac{\pi}{\Lambda}} \cdot \frac{2 \cdot \frac{\Lambda}{\pi}}{\sqrt{1 + \frac{\Lambda^2}{\pi^2}}}. \tag{10}$$

Comparing equations (9) and (5), and noting that $\log_e \rho = \Lambda$, it is seen that

$$R_2 = R_1 \frac{\rho}{1 + \rho}. \tag{11}$$

Inasmuch as the actual motion of the coil is the result of the superposition of the motions defined by (3) and (7), the exact equations of the two possible kinds of motion are obtained, respectively, first, by adding the two equations, and second, by subtracting the one from the other. The observed maximum then occurs not at the instant given by eq. (8), but a trifle sooner or later than this, depending upon the relative directions of the current and impulsive discharge. By equating to zero the angular velocities obtained by differentiation of each of the combined equations and solving for t, we may write the result for both cases:

$$t = \frac{1}{b}\tan^{-1}\frac{b}{a \pm \frac{i}{bQ}}. \tag{12}$$

The upper or lower sign is taken according as the current and discharge are in the same or in opposite directions, respectively.

It would be a rare occurrence in an ordinary measurement to encounter an extraneous thermoelectric current which is appreciable as compared with bQ, considered in its effect upon the time required for the throw. Should it occur, however, the procedure would be to find the correction by several approximations, using (12) instead of (8) in eq. (3). To substitute (12) directly in the equations of motion obtained by the addition or subtraction of (3) and (7), and then to form the ratios to be used for correcting the observed throws, would lead to expressions which are too complex for practical application.

EXPERIMENTAL.

Procedure in Correcting Observed Throws.—Consideration of equations (10) and (11) shows that for a determination of either R_1 or R_2 for any particular case a single experimental determination suffices, namely of the quantity ρ. The factors $\rho^{1/\pi \tan^{-1} \pi/\Lambda}$ and $\sqrt{1 + (\Lambda^2/\pi^2)}$ occurring in the equations are obtainable from tables.[1] This makes the formulas readily applicable. To use the correction given by eq. (10), the observed steady deflection due to the thermoelectric current flowing in the closed circuit is multiplied by R_1 and the result is added to or subtracted from the observed throw, depending upon the relative directions of current and discharge. To apply eq. (11), the first maximum amplitude due to the thermoelectric current only is observed, and the correction

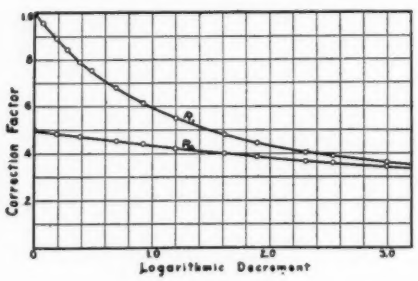

Fig. 1.

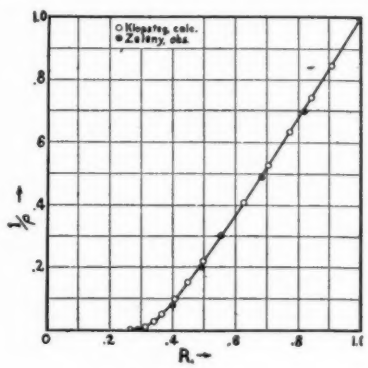

Fig. 2.

obtained by multiplying this value by R_2. Having once obtained R_2, its application is more economical of time than the use of R_1, since the first maximum elongation may be observed $T/2$ seconds after closing the circuit. This obviates the necessity of waiting for the deflection to attain its steady value, or of using a short-circuiting key for bringing the coil to rest.

The curves of Fig. 1 show the variation of each of the ratios with logarithmic decrement. In these curves the abscissas have been carried to $\Lambda = 3.2$ only, since, in practice, Λ seldom exceeds 1 when the galvanometer is used ballistically. If the logarithmic decrement is large, it is most convenient to render the coil just aperiodic by diminishing the resistance in the circuit. In the critically damped case $R_1 = R_2 = 26.4$ per cent. This is shown in Fig. 2.

Test of the Equations.—The experimental test of the method here outlined was taken from the work of Zeleny.[2] From eq. (10) a set of

[1] Kohlrausch, Lehrb. d. prakt. Phys., 12th ed., p. 723.

[2] Loc. cit.

corresponding values was obtained for R_1 and $1/\rho$, the abscissas and ordinates, respectively, of the experimental curve to which reference has been made. Fig. 2 shows the curve obtained by plotting these values, together with representative points taken from Zeleny's curve. There is good agreement between the theoretical and experimental curves.

Summary.

From a consideration of the equations of periodic motion of a galvanometer coil under the conditions, first, that a steady electromotive force exists in the circuit and second, that the electromotive force is zero, two expressions are deduced. Each expression gives the correction for thermoelectric current to be applied to the ballistic throw on closed circuit, the discharge having taken place on open circuit. The equations involve simply the determination of the ratio of one amplitude of the coil to the one next following, together with factors which are obtainable from existing tables. The calculated and observed corrections are found to be in good agreement.

Physical Laboratory,
The University of Minnesota,
February 2, 1916.

A RECORDING X-RAY SPECTROMETER, AND THE HIGH FREQUENCY SPECTRUM OF TUNGSTEN.

BY ARTHUR H. COMPTON.

ALTHOUGH the photographic method of obtaining the spectrum of a beam of X-rays reflected from a crystal has led to most interesting results in the hands of Moseley,[1] de Broglie[2] and others, it has been found possible to examine such a spectrum more thoroughly by ionization methods.[3] The reason for this is that while by the photographic method the intensity of the different spectrum lines can at best be only qualitatively measured, the ionization method is capable of giving quantitatively the relative intensity of the reflected beam of X-rays at different angles. The ionization method as usually applied, however, is open to the objection that a very large number of separate observations are necessary to obtain accurately a complete spectrum. In the present paper an apparatus will be described which gives a continuous record of the intensity of the beam of X-rays at different angles, and a study of the X-ray spectrum of tungsten will be made to illustrate the manipulation of the instrument.

This apparatus differs from the well-known Bragg X-ray spectrometer in two essentials: (1) The ionization current due to the reflected beam of X-rays, instead of charging up an electroscope directly, goes to one pair of quadrants of an electrometer and is shunted to earth through a high resistance. The electrometer thus acts as a highly sensitive galvanometer, a steady ionization current producing a steady proportional deflection. (2) The angle of the crystal and of the ionization chamber are varied continuously in such a manner that the ionization chamber is always in position to receive the reflected beam of X-rays, and the electrometer deflections corresponding to each particular angle of the crystal are recorded on a moving roll of photographic paper. In this manner a record of the complete spectrum can be obtained with a minimum of trouble and of exposure to the X-rays.

The arrangement of the apparatus is shown diagrammatically in Fig. 1. The X-rays pass from the anticathode A, through the slits B and B',

[1] H. G. J. Moseley, Phil. Mag., 26, 1024 (1913).
[2] M. de Broglie, Compt. Rend., 157, 924 and 1413 (1913).
[3] Cf. W. H. Bragg and W. L. Bragg, X-rays and Crystal Structure, p. 66.

are reflected from the crystal C and pass through the slits D' and D into the ionization chamber I. The ionization chamber rests on an arm fastened to the spectrometer table S, and the table on which the crystal rests is geared to move with half the angular velocity of the ionization chamber, though its position can also be varied by means of the slow motion screw F. The ionization current is carried from the chamber I to one pair of quadrants of a highly sensitive electrometer E, and is shunted to the ground through a variable xylol-alcohol resistance R. The intensity of the primary beam is measured in a similar manner by

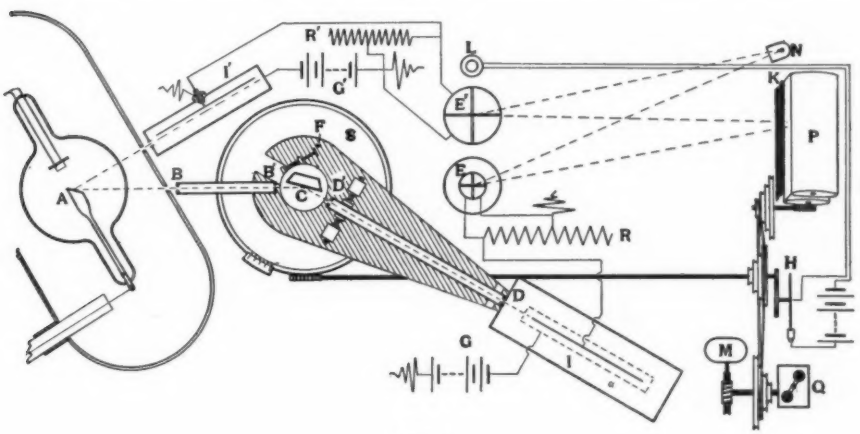

Fig. 1.

means of the ionization chamber I', the resistance R' and the electrometer E'. The mirrors of both electrometers reflect beams of light from the Nernst glower N, through a fine horizontal slit k onto a roll of bromide paper P. A motor-actuated driving clock MQ moves, by means of a system of pulleys and worm gears, both the spectrometer table and the roll of bromide paper at a constant, though adjustable, speed. The pointer H is so geared to the shaft which drives the spectrometer table that it makes one revolution for each degree through which the table turns, and by means of a mercury contact the lamp L is turned on for an instant at each degree or half degree. This illuminates the slit K and marks off the angle by a series of lines across the bromide paper, as is shown in the records, Figs. 4 to 11.

The construction of the system for measuring the reflected beam of X-rays is shown in more detail in Fig. 2. The ionization chamber I is closed at one end by a thin mica window m and at the other by a perforated brass cap covered with a plate of glass g. This arrangement allows the crystal to be adjusted optically to the angle of reflection.

The outside of the ionization chamber is grounded, but the wire framework f is raised to a sufficiently high potential to produce a saturation current to the wire w which is connected with the electrometer. The wire w passes through the ebonite plug e_1, is fastened to one terminal of the glass tube R containing a mixture of xylol and alcohol, and goes through the ebonite plugs e_2 and e_3 to the electrometer. The wire can be grounded directly by screwing up the mercury cup O until it touches a contact point, or it can be grounded through the desired high resistance by moving up one of the other mercury cups. If all the cups are down the system is insulated. The electric shielding consists of the rather heavy copper box c and the brass tube b. The electrometer E is placed

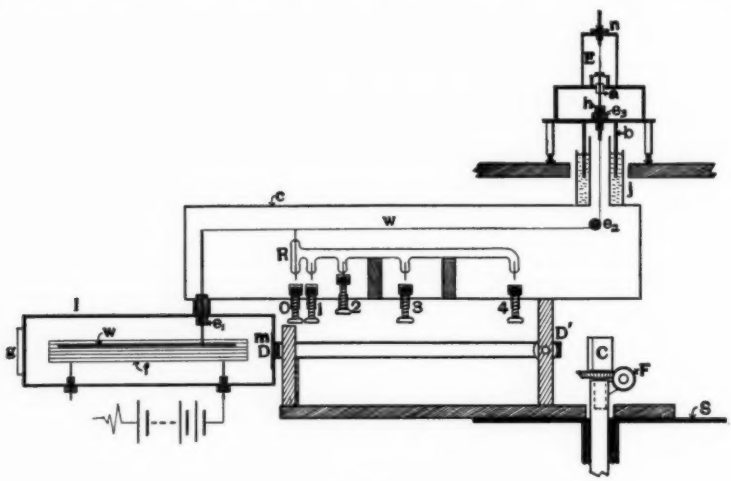

Fig. 2.

directly over the axis of the spectrometer, so that the joint in the shield at j permits free motion of the spectrometer arm, while an oil seal renders the shield airtight.

The electrometer E deserves particular mention, because it is so admirably adapted to this work. Its chief advantages lie in its flexibility, in its high sensitivity, in its perfect electrostatic shielding and in the use of a single piece of insulation for the sensitive quadrants. The quadrants of the electrometer are only 1.3 cm. in diameter. One pair of quadrants is supported by brass rods which fit into the electrometer case, and is thus permanently grounded; while the other pair is supported on a fine brass rod which passes through the amber insulation a and dips into the mercury cup h. The needle is suspended by a fine quartz fiber, and the whole system is sputtered with platinum. Most of the moment of inertia of the needle is due to the mirror, so this is made just large

enough to reflect a sufficiently strong beam of light. The needle is "dead-beat," and comes approximately to rest in a few seconds. With ten volts on the needle the electrostatic control is small, and with the suspension used the sensitivity is about 1,000 divisions per volt. When the potential of the needle is above forty volts, the electrostatic control becomes the determining factor, and the sensitivity can be varied from 1,000 divisions to ∞ by adjusting the position of the quadrants and the height of the needle. The highest sensitivity that it has been found practicable to use is about 25,000 mm. per volt at a distance of a meter. The capacity of the electrometer is about 15 electrostatic units, so that it is capable of measuring a very small charge.

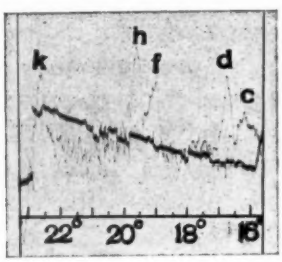

Fig. 3.

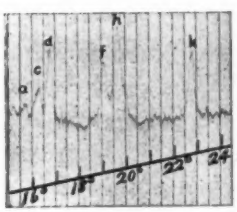

Fig. 4.

The Effect of Slight Radioactivity in the Ionization Chamber.—Fig. 3 shows an early attempt to obtain the X-ray spectrum of tungsten with this apparatus. The dark line represents the primary beam of X-rays and the light one the reflected beam. The electrometer in this case had a sensitivity of about 10,000 divisions per volt, and the shunting resistance was rather high. The electrostatic shielding at this time consisted of steel tubing and galvanized iron. Although the more prominent spectrum lines are easily distinguishable, there are so many spurious deflections of the electrometer which in no way correspond to variations of the primary beam of X-rays that it is impossible to make any accurate measurements. Supposing that these irregular motions were due to stray ionization currents within the electrostatic shield, a copper and brass shield was used instead, thus decreasing the contact potential difference between the shield and the wire. In this manner the relative size of the spurious deflections was much reduced, as is shown in Fig. 4, though these motions were still so large as to interfere seriously with the readings.

Investigation showed that the remaining irregular motions of the electrometer needle were due, in large measure at least, to real ionization

currents in the ionization chamber, which were independent of the X-rays, as is shown in Fig. 5 in which the X-ray tube was not running. The magnitude of this current was found to be of the same order as the natural ionization current usually observed in brass vessels. It was also noticed that the deflections always occurred suddenly in the direction of the normal ionization current, and then died out gradually, which suggested ionization due to alpha particles.

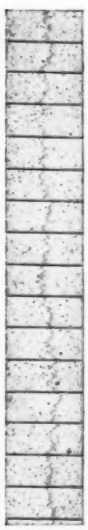

Fig. 5.

As will be shown later (equation 4 below), if the shunt resistance and the sensitivity of the electrometer are known, the total ionization corresponding to each hump on a record can be calculated from the area under the hump. When such a calculation is made on the record shown in Fig. 5, it is found that on the average each hump corresponds to the production of about 8×10^4 ions. Not all of the alpha particles will escape from the walls of the ionization chamber with their full velocity, since most of them will start at some distance from the surface of the metal. If we assume that the alpha particles have a definite range in the metal, a simple calculation shows that the average range of the emitted particles will be half the maximum range. Since, however, the ionization is somewhat greater per centimeter path for the more slowly moving particles, the average ionization will be somewhat more than half, about 0.6 of that produced by a particle which starts from the surface (assuming the emitted particles to be completely absorbed by the gas in the ionization chamber). A particle starting from the surface therefore produces about $8 \times 10^4/0.6 = 1.3 \times 10^5$ ions. An alpha particle from radium would produce 1.5×10^5 ions, which agrees well enough with the observed value to show that these spurious deflections are doubtless due to alpha particles given off either by the walls of the ionization chamber or by the gas within it.[1]

[1] An effect exactly similar to that here described has been observed by Rutherford and Geiger (Proc. Roy. Soc., *A*, 81, 141 (1908)) while counting the alpha particles from radium by an ionization method. They attributed the effect to the natural radioactivity of the metal of which the ionization chamber is composed. Recently Shrader (PHYS. REV., 6, 292 (1915)) has found that if the electrode within the ionization chamber has a sharp point, and if the chamber is at high enough potential to produce ionization by collision, the "natural disturbances" depend only on the nature of the point, and can be practically eliminated if the point is properly treated. It is to be noticed, however, that the only disturbances which would affect his electroscope would be the ones which would ionize the air near the needle point of his electrode, where any ionization was greatly magnified by collision. It would seem that his apparatus was not sufficiently sensitive to be affected by the occasional production of alpha particles in other parts of the ionization chamber. His work cannot, therefore, be taken to indicate that the walls of his ionization chamber were not slightly radioactive, so that his results are not inconsistent with the writer's and those of Rutherford and Geiger.

A thorough cleaning of the ionization chamber with nitric acid seemed to reduce the amount of radioactivity but little. Its relative effect could be greatly diminished, however, by filling the ionization chamber with a dense gas, and thus increasing the ionization due to the X-rays. The spectrum shown in Fig. 6 was obtained in this manner, using a simple ionization chamber such as I', Fig. 1, filled with dry hydrogen iodide. The effect of the alpha particles was still fruther reduced by changing the ionization chamber to the type shown in Fig. 2. The ionization due to the X-rays is produced chiefly within the wire network f, which is kept at a sufficiently high potential to produce a saturation current to the wire w. The outer casing of the ionization chamber is grounded, and enough air-space is left between the casing and the network to absorb all the alpha particles which leave the walls. Thus the particles from the walls of the vessel do not affect the current flowing to the wire w; this is affected only by the radioactivity of the wire netting and of the gas within it. By this means the number of spurious deflections of the electrometer was reduced to about 1/4 of the number when the simpler ionization was used, while the ionization due to the X-rays remained about the same. All of the spectra after Fig. 6 were obtained with this new ionization chamber.

A Study of the High Frequency Spectrum of Tungsten.—As a source of X-rays for the examination of the spectrum of tungsten, a Coolidge X-ray tube with a tungsten anticathode was used. In order that the tube should remain steady over long runs it was found best to heat the thermionic filament by means of batteries of rather large capacity, 175 ampere hours. The source of high potential was a Snook-Roentgen machine. This consists of a 10-kilowatt step-up transformer with a commutating device in the secondary circuit, which makes it possible to obtain a direct though intermittent high potential current. This apparatus was easily capable of running the Coolidge tube at its maximum capacity, and was found to be very satisfactory. In most of the runs a current of about 30 milliamperes was sent through the tube, with an alternate spark gap between sharp points of about 10 cm.

Fig. 6 shows a complete spectrum of the X-radiation from tungsten as analyzed by a crystal of zinc-blende. As in the other figures the heavy line indicates the strength of the primary beam, and the light one of the reflected beam of X-rays, though of course the two curves are on greatly different scales. Each vertical line corresponds to one degree on the spectrometer table, or 1/2 degree of rotation of the crystal. The angles marked on all of the spectra indicate the glancing angle θ of the beam of X-rays which strikes the crystal. The broken base line

represents the zero point for the electrometer measuring the reflected beam, and the solid line that of the one measuring the direct beam of X-rays. At the angle $\theta = 30.5°$ a zero reading was taken, by putting a lead screen between the slit B' and the crystal, to make sure that the zero point of the sensitive electrometer had not shifted. In obtaining this spectrum wide slits were used, the slit at D being about 1.0 cm. The shunting resistance was about 3.5×10^{11} ohms, and the sensitivity 1,050 mm. per volt. With this low resistance and wide slits it was possible to go over the complete spectrum in an hour and a half.

Owing to the comparatively great wave-length of the characteristic L-radiation from tungsten and to the closeness of the layers of atoms in

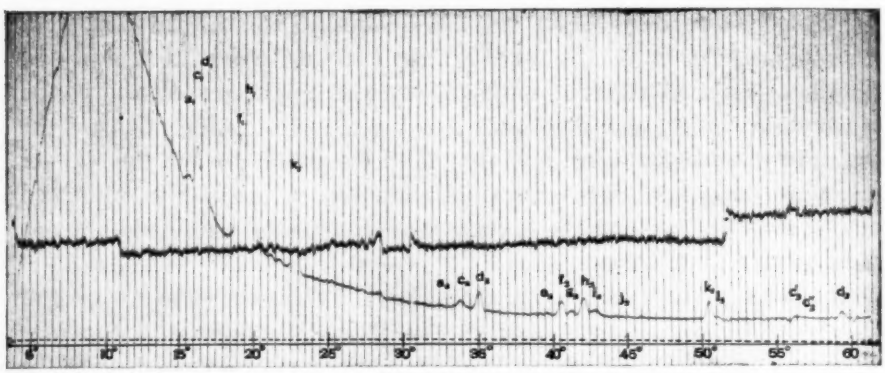

Fig. 6.

the cleavage planes of zinc-blende, it is impossible to obtain a complete third order spectrum in this case, but the first and second orders stand out very clearly. The line marked k is the single line observed in the spectrum of tungsten by Moseley,[1] and the prominent lines c, d, f, h and k correspond to the five lines observed by Moseley and Darwin[2] in the spectrum of platinum. Recently Barnes[3] has detected also the lines g and i in the tungsten spectrum by a photographic method. Lines at a and l can also be seen plainly in this photograph, l being visible only in the second order spectrum because of its proximity to the prominent line k. An indication is also given of possible lines at e and j. These are confirmed by other records, though the line j is rather doubtful. A series of four records, such as Fig. 7, which is a part of the first order spectrum from rock-salt using a slit at D only 0.4 mm. wide, indicates that the line c is really a very close double. A line also appears con-

[1] H. G. J. Moseley, Phil. Mag., 27, 703 (1914).

[2] H. G. J. Moseley and C. G. Darwin, Phil. Mag., 26, 211 (1913).

[3] J. Barnes, Phil. Mag., 30, 368 (1915).

sistently on a number of records between *a* and *c*. All of these lines are shown more or less distinctly in Fig. 8.

In order to make an accurate determination of the wave-length of the different lines, a crystal of calcite was used, as recommended by Professor Bragg, because of the extremely perfect faces obtainable. The X-ray tube was swung around until the rays which fell on the crystal left the target almost tangent to its face, the slits *B* and *D* were about 2 mm., and the slits *B'* and *D'* were made wide. The slit at *D* and the target of the X-ray tube were set as nearly as possible at equal distances, about 52 cm., from the crystal. With this arrangement it was found possible to measure the angles of reflection with considerable precision.

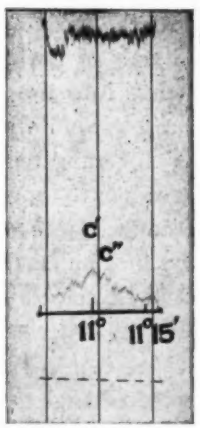

Fig. 7.

It was necessary first to determine the zero point accurately. This was done by taking a series of four records, two on either side of the zero point, one with the spectrometer moving toward and the other from zero. Fig. 8 is one of this series, taken with the spectrometer moving toward the zero point. The mean angle as measured from the three most prominent lines in the four records was taken as the true zero, and this could be determined with a probable error of less than ± 0.2′. The angle of the

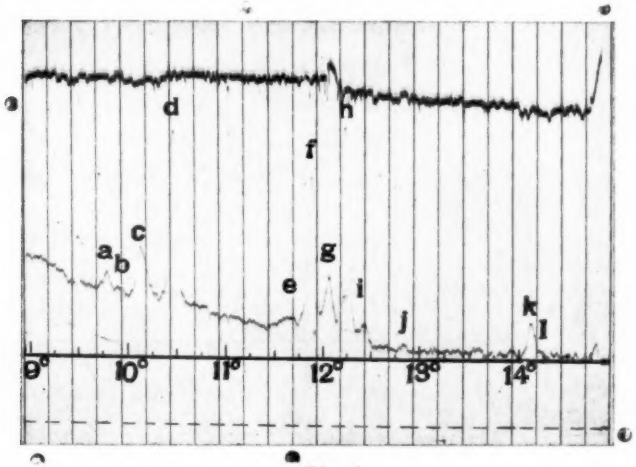

Fig. 8.

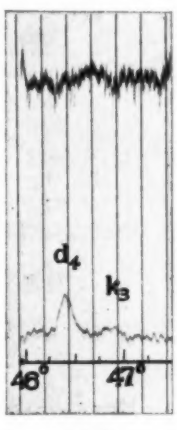

Fig. 9.

spectrum lines was then determined from a series of six records of the first order spectrum and two records showing the third order *k* and the fourth order *c*, *d*, *f* and *h* lines. Fig. 9 shows a part of one of these

records, including the fourth order d and the third order k lines. The angles found for the higher order spectra did not correspond exactly with those calculated from the first order according to the formula

$$n\lambda = 2d \sin \theta.$$

The reason for the difference was traced to a combination of two causes: first, the front edge of the crystal projected about 0.5 mm. in front of the axis of the crystal table, and second, the slight penetration of the X-rays into the crystal made the observed angle slightly less than the true angle of diffraction. The latter effect is very small, being only about 0.2′ for tungsten rays diffracted by calcite. For X-radiation of short wave-length, however, and a crystal such as diamond, this error becomes of considerable magnitude, as has been noticed in the case of rhodium rays.[1] The correction due to the inaccurate adjustment of the crystal could be calculated accurately from a comparison of the position of the different orders of the same line. The sum of the two corrections was 3.15′ ± 0.10′, being practically equal for 'the different lines. The corrected angles of the lines of the first order spectrum are given in column 2 of Table I. The error in the angle here given for the more intense lines

TABLE I.

Line.	θ	$\lambda \times 10^8$ Cm.	Barnes's Values.	Remarks.
a	9° 44.5′	1.0387		
b	9° 53.1′	1.0539		Line somewhat doubtful.
c'	10° 3.7′	1.0725	1.082	Very close double.
c''	10° 7.7′	1.0796		
d	10° 25.7′	1.1107	1.113	
e	11° 36.3′	1.2349		
f	11° 49.9′	1.2587	1.258	
g	12° 0.4′	1.2771	1.277	
h	12° 11.4′	1.2962	1.296	
i	12° 22.7′	1.3160	1.312	
j	12° 44.7′	1.3543		Line uncertain.
k	14° 4.7′	1.4933	1.477	Moseley gives 1.486.
l	14° 11.1′	1.5044		

is probably less than ± 0.5′, though for some of the fainter lines the probable error is as much as ± 1.0′.

The distance between the successive atomic layers in the cleavage planes of calcite may be calculated according to the formula[2]

[1] W. H. Bragg, Phil. Mag. 27, 898 (1914).

[2] W. H. Bragg, Proc. Roy. Soc., *A*, 89, 468 (1914). "X-rays and Crystal Structure," p. 112. Professor Bragg uses the value $\phi(\beta) = 1.08$, which makes his value of d for calcite differ appreciably from that here obtained.

$$d = \left(\frac{1}{2}\frac{M}{\rho N \phi(\beta)}\right)^{\frac{1}{3}},$$

where M is the molecular weight of $CaCO_3$, ρ is the density of the crystal, N is the number of molecules per gram-molecule, and $\phi(\beta)$ is the volume of a rhombohedron the distance between whose opposite faces is unity and the angle between whose edges is β. This function may be shown to be

$$\varphi(\beta) = \frac{(1 + \cos\beta)^2}{\sin\beta(1 + 2\cos\beta)}.$$

For calcite, β is 101° 54′, which makes $\phi(\beta) = 1.0960$[1]. The density of the crystal used was carefully determined, and was found to be $\rho = 2.7116 \pm 0.0004$ g. cm^{-3}. (at 18°). Taking $M = 100.09$ and $N = 6.062 \times 10^{23}$ (Millikan), we find $d = 3.0279 \times 10^{-8}$ cm. From this value of d the wave-lengths can be immediately calculated according to the expression

$$\lambda = 2d \sin\theta.$$

Using the values of θ given in the second column of Table I., we obtain the wave-lengths given in the third column. For convenience in comparison the wave-lengths found by Barnes[1] are given in the fourth column.[2]

The Intensity of the Different Orders of Spectra.—It is difficult to obtain an accurate estimate of the relative intensity of the various lines in the same order of the spectrum, on account of the difference in the absorption of the corresponding wave-lengths. It is possible, however, to make a comparatively accurate determination of the relative intensity of the different order reflections of the same line. In order to do this, the slit at B' was made narrow, about 0.9 mm., slit D' was removed, and slit D was made 12 mm. wide, which was more than broad enough to take in all the rays reflected from the crystal. When the crystal and ionization chamber are moved at uniform speed past a spectrum line, the area under the curve representing the line on the resulting record is proportional to the integral of the ionization current produced by the line, and hence to the intensity of the line itself. Thus by comparing

[1] J. Barnes, loc. cit.

[2] Note added May 12, 1916: In a preliminary report on this work, made at the meeting of the American Physical Society on February 26, the wave-lengths of the spectrum lines were calculated using as the grating space for calcite 3.0695×10^{-8} cm. instead of the value here found. The results thus obtained agreed acceptably with Barnes's measurements. W. S. Gorton has since shown (PHYS. REV., Feb., 1916), by comparison with rock-salt, that d for calcite is rather about 3.028×10^{-8} cm., which called my attention to an error in my previous calculation of the grating space. The wave-lengths here obtained are in much better agreement with those found by Gorton than with those due to Barnes.

the areas under the humps on the record, the intensities of the different order lines may be compared.

Since the period of the needle and the time required for the electrometer to charge up to its maximum potential might also be expected to affect the area of the hump, it may be worth while to investigate the question in detail. The equation of motion of the spot of light reflected by the electrometer needle onto the photographic paper is

$$I\frac{d^2y}{dt^2} + D\frac{dy}{dt} + My = \frac{M}{a}V, \tag{1}$$

where y is the deflection of the spot of light, I is the inertia term, D the damping term, M represents the restoring force, $1/a$ is the sensitivity of the electrometer, and V is the potential difference between the two pairs of quadrants. On integrating this expression between the times t_0 and t_1 we obtain,

$$\frac{Ia}{M}\left[\frac{dy}{dt}\right]_{t_0}^{t_1} + \frac{Da}{M}\left[y\right]_{t_0}^{t_1} + a\int_{t_0}^{t_1} ydt = \int_{t_0}^{t_1} Vdt. \tag{2}$$

The potential of the system at any time is given by the expression

$$V = Ri - RCdV/dt,$$

where R is the resistance of the shunt to ground, i is the ionization current, and C is the electrostatic capacity of the system. Integrating as before,

$$\int_{t_0}^{t_1} Vdt = R\int_{t_0}^{t_1} idt - RC\left[V\right]_{t_0}^{t_1}.$$

Combining with equation (1) and substituting in (2) we get

$$\begin{aligned}\int_{t_0}^{t_1} idt = \frac{a}{R}\int_{t_0}^{t_1} ydt + \left(\frac{aD}{RM} + aC\right)\left[y\right]_{t_0}^{t_1} \\ + \left(\frac{aI}{RM} + \frac{aCD}{M}\right)\left[\frac{dy}{dt}\right]_{t_0}^{t_1} + \frac{aCI}{M}\left[\frac{d^2y}{dt^2}\right]_{t_0}^{t_1}.\end{aligned} \tag{3}$$

Since the intensity of a line is proportional to the total ionization produced in the ionization chamber as it moves past the line, expression (3) is evidently a measure of the intensity if the times t_0 and t_1 are taken just before and just after the line is traversed. If the displacement, velocity and acceleration of the spot of light are the same at the times t_0 and t_1, equation (3) reduces to

$$\begin{aligned}\int_{t_0}^{t_1} idt = \frac{a}{R}\int_{t_0}^{t_1} ydt = \frac{a}{R}\frac{dt}{dx}\int_{x_0}^{x_1} ydx \\ = \frac{a}{R}\frac{dt}{dx}A\ ,\end{aligned} \tag{4}$$

where dx/dt is the velocity with which the roll of paper moves, and A is the area under the hump on the record. In practice it is easily possible to satisfy these conditions with sufficient accuracy, so that the intensity may be taken to be proportional to the area under the curve.

The records shown in Figs. 10 and 11 are examples of this method of comparing intensities. Fig. 10 shows the first and second order f and

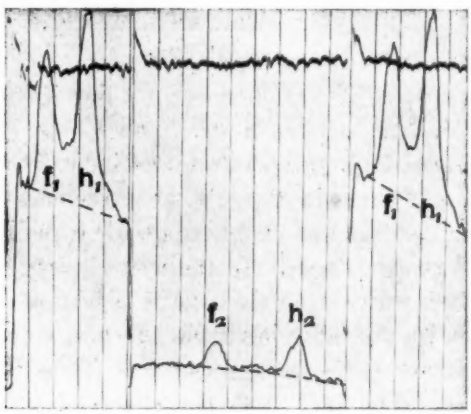

Fig. 10.

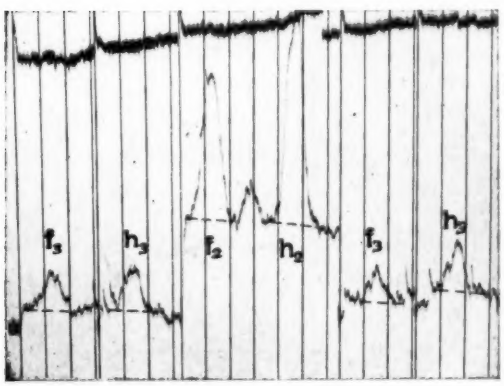

Fig. 11.

h lines as reflected from a cleavage plane of rock-salt, and in Fig. 11 the second and third orders of these lines are compared. In Fig. 10 the lighter line has been inked, in order to make a better reproduction. The areas under the humps were measured with a planimeter. The largest part of the error probably occurred in selecting the base line from which

to measure the area. These records give the ratio of the intensities of the three orders as 100 : 18.9 : 3.9.

Bragg[1] has studied the relative intensities of the first three orders of reflection from rock-salt, using the rhodium line of wave-length 0.614 $\times 10^{-8}$ cm. He found that he could express his results within the probable error of his experiments by the formula

$$J_\theta = \frac{A(1 + \cos^2 2\theta)\, e^{-\beta \frac{\sin^2 \theta}{\lambda^2}}}{\sin^2 \theta},$$

where J_θ is the intensity of a given spectrum line when reflected at the angle θ, and A and B are constants. The factor $(1 + \cos^2 2\theta)$ was deduced theoretically by J. J. Thomson, and indicates a polarization of the reflected beam. Since in Bragg's measurements θ never became greater than about 20°, he was unable to decide experimentally whether this factor should really appear. In the writer's experiments the angle θ for the third order was 43°, so that the polarization should have been nearly complete. In the following table the first calculated values do not include the factor $(1 + \cos^2 2\theta)$, but this factor is included in the second calculation. The fact that the observed values agree much

TABLE II.

Order.	Mean Angle.	Intensities.		
		Calc.$_1$	Obs.	Calc.$_2$
1	13° 5′	100.0	100.0	100.0
2	26° 55′	21.6	18.9	16.2
3	42° 59′	7.5	3.9	4.2

more closely with the values calculated when this factor is considered shows that the crystal grating does polarize the reflected beam and that this polarization is approximately complete when the angle of reflection is 45°.

Summary.—A recording X-ray spectrometer has been described by means of which a continuous record can be obtained of the ionization produced by a beam of X-rays reflected from a crystal as the angle of the crystal is varied.

A spurious and irregular current in the ionization chamber has been observed, which has been found to be due to a slight radioactivity within the ionization chamber. Methods are described whereby the effect of this radioactivity may be considerably reduced.

[1] W. H. Bragg, Phil. Mag., 27, 893 (1914). also "X-rays and Crystal Structures," p. 198.

The X-ray spectrum of tungsten has been examined in some detail. In addition to the seven lines already known, an indication has been found of six others, two of which are, however, somewhat uncertain. The wave-lengths of the different lines have been carefully determined.

The relative intensities of the different order reflections of the same line have been shown to be proportional to the areas under the humps produced in the record by the spectrum line; and the relative intensities of the first three orders in the spectrum from rock-salt have been determined by measuring these areas.

The observed intensities of the different orders indicate that polarization occurs when a beam of X-rays is reflected by a crystal.

In conclusion I wish to thank Professor H. L. Cooke for many valuable suggestions in the design of the apparatus and for helpful advice in all parts of the work.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
February 16, 1916.

A NEW METHOD FOR ENLARGING PHOTOGRAPHS WITHOUT THE USE OF A LENS.[1]

BY ALFRED J. LOTKA.

AN observation of a peculiar distortion presented by the appearance of certain moving objects led the writer to the development of the method of enlarging photographs described below.

This method consists in moving the negative to be enlarged past a narrow slit source of light, and at the same time moving a sensitive plate under the negative at a speed equal to some constant multiple n of the speed of the negative.

On development of the sensitive plate a positive transparency is obtained, in which all lines of the original negative which were parallel to the slit during the exposure are unaltered, while all lines at right angles to the slit are magnified in the ratio $n : 1$.

This positive is subsequently subjected to a repetition of the process practised on the original negative, but with the motion now at right angles to the lines drawn out n-fold in the first operation. The result of the second operation is a negative geometrically similar to the original, but with the linear dimensions enlarged n times, or the area n^2 times.

The apparatus employed for carrying out the process has been constructed on exceedingly simple lines, with the use of material readily obtainable on the market. The slit is made with two strips of brass picked from the stock of a wholesale hardware store and having received no other treatment than a cleansing with a brass polish.

The ratio n of the velocities in the apparatus so constructed is 2, and is secured by the simple expedient shown diagrammatically in Fig. 1. In this drawing the original negative is represented at A, the sensitive plate at B, and the slit at S. The carrier C, on which the plate B rests, has attached to it at each end a silk thread D, which passes through an eye E, attached (with gummed paper) to the glass plate of the negative (when a celluloid film is used, such film may be attached with gummed tape to a glass plate). The further end of the silk cord D is attached to the end block of the box containing the negative and plate during exposure. The carrier C slides on the bottom of the box, between guides adjusted to fit the sides of the plate.

[1] Demonstrated at the New York meeting of the American Physical Society, February 26, 1916.

It will readily be seen that, in accordance with the principle of the single movable pulley, the velocity of the sensitive plate is always double that of the negative.

Any means, such as a hand-pull at P, may be employed to impart uniform motion to the negative. A cylinder containing water through which a fairly snug-fitting perforated piston is drawn makes a good dash-pot to regulate the motion. This and a still simpler and exceedingly compact device for this purpose is a subject of a pending patent application.

The adjustment of the lamp vertically above and in the plane of the slit is effected by looking down at the lamp and its reflection in the glass covering the slit. When the filament, its image, and the line of the slit coincide in the line of sight, the filament is in the plane of the slit. It was found that with a slit 6½ inches long good results were obtained with a filament of six or seven coils placed about one foot from the slit.

Among the advantages which the new method presents is uniformity of illumination over the entire field; geometric similarity to the original of the image produced, *i. e.*, absence of distortion such as spherical aberration introduces; compactness and simplicity of apparatus, replacement of lens by a simple slit and hence low cost of the apparatus.

The process necessitates two successive exposures, but this is true also of the ordinary method when a negative is to be prepared from a negative.

The example shown in the accompanying half-tone illustration gives an idea of the working of the process, though it does not represent the best that can be done in the way of definition.

The limiting separation which the apparatus will effect can be figured as follows:

Consider a point right on the edge of the negative. This will begin to be reproduced on the sensitive plate as soon as the negative and plate are in the position shown in Fig. 2*a*. (The slit is here shown on a greatly enlarged scale.) The same point will continue to be reproduced until negative and sensitive plate are in the position shown in Fig. 2*b*. It will therefore be drawn out into a line of length $ns - s = (n - 1)s$.

Now consider two points, a distance d apart in the negative. In the copy they will appear as two lines $(n - 1)s$ long and with their centers nd apart. These two lines will just merge at their ends when

$$nd = (n - 1)s,$$

$$d = \frac{n - 1}{n} s.$$

This distance

$$d = \frac{n - 1}{n} s$$

between two points on the original negative represents the limit of separation: Two points a distance greater than d apart will be separated, two points nearer together than d will not be separated.

For $n = 2$, as in the apparatus constructed, we have

$$d = \tfrac{1}{2}s.$$

Thus, for example, a comparatively coarse slit 1/100 inch wide, will still separate points just a little over 1/200 inch apart.

The process and apparatus here described are protected by U. S. Pat. 1,176,384 of March 21, 1916, issued to the writer.

GENERAL CHEMICAL COMPANY,
NEW YORK.

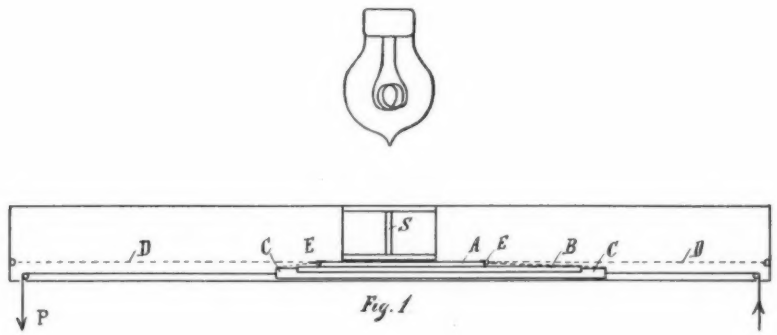

Fig. 1.

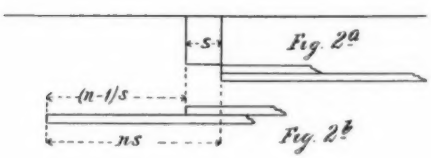

Fig. 2.

Figs. 1 and 2. Diagram of Apparatus.

Fig. 3.

Half-tone copy of original photograph.

Fig. 4.

Copy of distorted positive transparency obtained in first step of process.

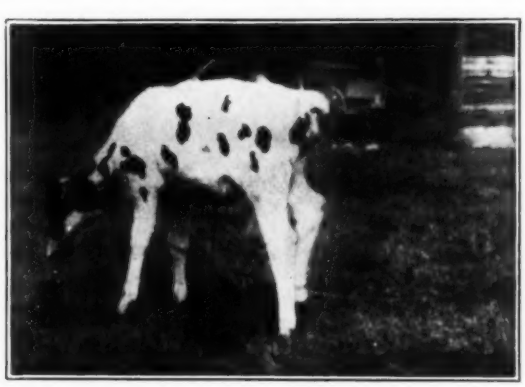

Fig. 5.

Copy of enlargement obtained in second step.

ALFRED J. LOTKA.

THE EFFECT OF A MAGNETIC FIELD ON THE CONDUCTIVITY OF FLAMES.

By C. W. Heaps.

WHEN an ionized gas is acted upon by an electric force the positive and negative ions drift in opposite directions and an electric current is produced. The velocities which the ions acquire under the influence of the electric field have been determined under different conditions and in a variety of ways. It is found that for small electric fields the velocity of the negative ion is proportional to the field and is in general greater than the velocity of the positive ion.

When the ionized gas is a Bunsen flame the negative ion appears to acquire a much greater velocity than the positive ion, and experiments which have been made give somewhat different values for this velocity. By a study of the Hall effect a value of 2,450 cm. per sec. for one volt per centimeter has been obtained;[1] measurements of conductivity under direct and alternating fields gave velocities varying from 5,300 to 12,900;[2] measuring the effect of a magnetic field on the resistance of a flame gave for the above quantity a value of about 5,000 cm. per sec. for one volt per centimeter.[3]

In the experiments described below the velocities of negative flame ions are determined for different values of the electric intensity and it is shown that when small electric forces act the velocity under unit force is smaller than that when larger forces act. Townsend has shown[4] that for gases at ordinary temperatures the velocity of the negative ion may be a somewhat complicated function of X/p, where p is the pressure of the gas and X is the electric intensity. When X/p is small the velocity varies as X/p; when X/p is between 0.01 and 0.2 the velocity increases rapidly; and when X/p is large the velocity increases more slowly than X/p. The experiments described below indicate that the velocity of the negative ions in a flame varies with X/p in a manner similar to that found in gases at ordinary temperatures. For small values of X/p the velocity

[1] H. A. Wilson, Phys. Rev., III., p. 375, 1914.

[2] H. A. Wilson, Electrical Properties of Flames. E. Gold, Proc. Roy. Soc., 79, p. 43, 1907.

[3] H. A. Wilson, Proc. Roy. Soc., 82, 1909. The above velocity is obtained by using the formula derived in the latter part of the present paper, and not by the formula of Wilson's paper.

[4] J. S. Townsend, Electricity in Gases.

is proportional to X/p; a stage is reached when the velocity increases rapidly with X/p; and for large values of X/p it becomes again nearly proportional to X/p.

The experiment consisted in determining the change of conductivity (which is defined as the ratio of the current to the potential gradient) produced in a flame by the presence of a magnetic field. In a similar experiment performed by H. A. Wilson,[1] in which a vertical flame and a horizontal current were used, complications were found to enter, probably because of the upward motion of the flame gases. In the present experiment, therefore, the electric current was made to flow vertically, in the same direction as the flame gases.

The flame was produced by burning a mixture of gasoline vapor and air flowing through a special type of burner. The air and the gas were well mixed in the proper proportions before entering the burner, the air coming from a reservoir where it was maintained at constant pressure. The burner consisted of a vertically mounted brass tube, 2.4 cm. in diameter, into the top of which an iron grid was fitted. This grid caused the ordinary green cone of the Bunsen flame to be broken up into a large number of very small cones. A large volume of homogeneous flame was thus available above the green cones for experimenting purposes. A brass tube 3 cm. in diameter and 6.5 cm. long was fitted loosely around the burner so as to project 1.2 cm. above the grid. This tube tended to make the flame steady. When the air and gas are mixed at some distance from the flame this type of burner gives a much steadier flame than the ordinary Bunsen burner, and in making measurements of conductivity steadiness is desirable if accuracy is to be secured.

The electrodes by which the current entered and left the flame were of platinum wire mounted on ebonite rods and bent to form a coarse grid about one centimeter in diameter. They were in series with a shunted galvanometer, sensitive to 3.8×10^{-8} amperes, a reversing switch, and a battery of small dry cells.

The potential difference between any two points in the flame could be measured by means of two platinum exploring wires connected to opposite quadrants of a Dolezalek electrometer. In order to secure steadiness of the electrometer the needle was equipped with a small rod projecting downward between the quadrants and immersing itself in a small insulated pot of castor oil. The electrometer gave a deflection of about 3 divisions per volt, and was calibrated by comparison with a Weston voltmeter.

The magnetic field was produced by a Weiss electromagnet with flat

[1] Proc. Roy. Soc., 82, 1909.

pole-pieces 10 cm. in diameter and 5.1 cm. apart, between which the flame was placed. The magnetic field was measured with a bismuth spiral. A diagram of the apparatus is given in Fig. 1, where G is a galvanometer, E the electrometer, and F a fuse of aluminium foil.

Experiment has shown[1] that when a current flows through a flame between platinum electrodes there is a large potential drop near the cathode and a small uniform gradient in the rest of the space. Covering the cathode with a coat of lime[2] or salt[3] causes this cathode fall of potential to disappear, the current through the flame is increased, and the previously small uniform gradient is made larger. When vertically placed electrodes are used in a flame it is necessary to make the top electrode the cathode if it is to be coated with salt or lime, otherwise the flame between the electrodes becomes filled with particles of salt or lime and its conductivity is changed.

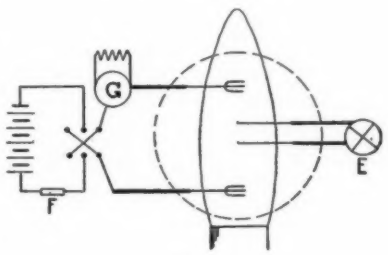

Fig. 1.

In the present experiment the top electrode—the cathode—was covered with the deposit formed by burning ordinary sealing wax on the platinum terminal. This deposit, consisting of metallic oxides, was found to produce the same effect on the potential gradient as a salt layer, and it had the added advantage of being much more lasting.

If the density of the ions varies from one part of the flame to another the conductivity will be different in different parts also. This variation in conductivity arising from change of position was examined by moving the exploring terminals of the electrometer along the flame between the electrodes and measuring the change of potential gradient. When the electric field was small the conductivity was found to be higher in the lower part of the flame. In one case it decreased about four times in moving from a point near the anode to a position not far from the cathode. For larger electric fields there was not such a large variation, and furthermore the conductivity was higher in the upper part of the flame near the cathode. When a transverse magnetic field acted the variation of conductivity was in general made greater.

The curves of Figs. 2, 3, 4, and 5 show the results of these measurements when different voltages were applied to the electrodes. The number on each curve is the value of the applied potential for which the curve was obtained. The exploring terminals of the electrometer

[1] H. A. Wilson, Phil. Trans., *A*, 192, p. 499, 1899.

[2] Tufts, Phys. Zeitschr., Vol. V., p. 76, 1904.

[3] H. A. Wilson, Electrical Properties of Flames.

were 0.8 cm. apart and the current electrodes were 4.8 cm. apart. The solid line gives the potential gradient when the magnetic field is zero and the broken line the gradient for a field of 3,600 gauss. It will be observed that the broken line always crosses the solid line—it must do this since the average potential gradient over the whole region between the electrodes is not changed appreciably by the magnetic field.

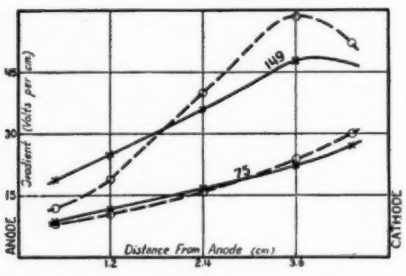

Fig. 2.

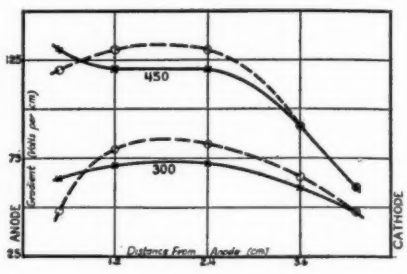

Fig. 3.

The curves shown cannot be taken as representing the exact variation of potential gradient, because if this gradient changes very rapidly electrometer terminals 0.8 cm. apart will not give the correct slope of the curve. A point on any curve gives the average potential gradient over a distance of 0.8 cm. When the exploring points were moved more

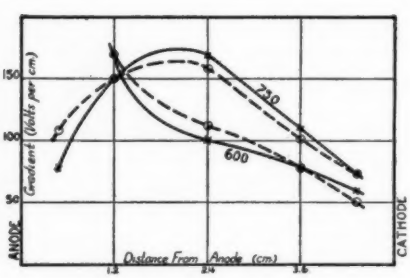

Fig. 4.

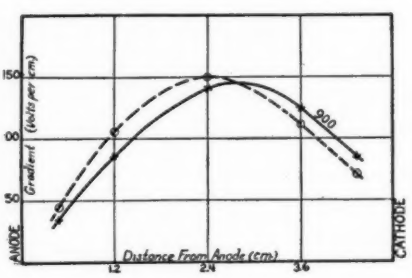

Fig. 5.

closely together the existence of a fairly large decrease of potential gradient was detected near the cathode when 149 volts were applied.

The electrometer was insensitive for very large potential differences. It would not measure a gradient larger than 160 or 170 volts, and points on the curve above 150 are not reliable. In view of this fact some observations were made with applied potentials of 450 and 600 volts, using a multicellular electrostatic voltmeter reading to 180 volts in place of the electrometer. When one terminal of this voltmeter was connected to the anode and the other to a single exploring point in the flame a

large drop of potential was detected near the anode for the above applied potentials. With 600 volts there was a drop of over 180 volts inside of a millimeter distance from the anode, whereas the electrometer indicated a much smaller gradient. With smaller applied potentials this anode drop became much smaller.

It is evident, therefore, that the midpoints of the curves for 750 and 900 volts cannot be relied upon as being very exact. The electrometer did not give very accurate readings for the high points of the curves. In the curve for 900 volts a large value of the potential gradient must exist either at the electrodes or in the middle region, otherwise the average height of this curve would be less than that of the curve for 750 volts. This, of course, cannot be the case, so that these curves must be taken to represent the general way in which the gradient changes at some distance from the electrodes.

The curves of Figs. 2, 3, 4, and 5 are of interest because they indicate in a general way how the electric charge in the flame varies with applied potentials. The equation

$$\frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} = -4\pi\rho$$

becomes in this case, approximately

$$\frac{dX}{dx} = 4\pi(n_2e_2 - n_1e_1),$$

where n_1 and e_1 are the number of positive ions per cubic centimeter and the charge on each, respectively; n_2 and e_2 are the corresponding quantities for the negative ions. From this equation we see that if the slope of the curve is positive there will be a negative charge in the flame. Inspection of the curves shows that for small applied potentials there is a negative charge in the flame, but that for potentials over 300 the charge becomes positive. This fact is in agreement with the idea that a fairly large field will remove the more mobile negative ions at a greater rate than the positive ions. It is also probable that the anode, when raised to a high positive potential, will remove negative ions from the gas streaming up through it.

For very large applied potentials (750 and 900 volts) a negative space charge appears near the anode. The reason for this charge will be clearer after a consideration of Fig. 6.

Fig. 6 shows the relation between the current through the flame and the applied potential, both with and without a magnetic field. The current was measured by the deflection of the galvanometer, which was shunted with a resistance of 20 ohms. The seven points on each curve

correspond to the seven values of the applied potential for which Figs. 2, 3, 4, and 5 were obtained. Several other curves of the same type as those of Fig. 6 were obtained, and they all showed the same general characteristics. They resemble very much the ordinary curves obtained with gases at low temperatures when saturation and ionization by collision are obtained. In the case of the flame, however, a true saturation current for the whole region between the electrodes is not obtained when the curve approaches parallelism with the voltage axis. The existence of a larger drop of potential near the anode than in the rest of the flame

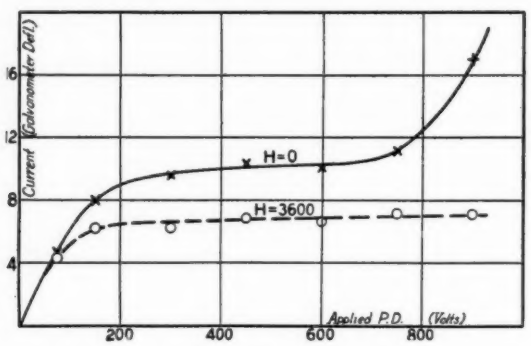

Fig. 6.

indicates that the rest of the flame has a relatively large number of negative ions present. Moreover, the appearance of an anode fall of potential will explain the observed facts as follows:

The current through the flame has been found[1] to obey the equation

$$V = Acd + Bc^2,$$

where V is the applied potential, c is the current, and d the distance between electrodes. A and B are constants. The term Bc^2 represents the potential drop near the electrodes, and Acd the drop in the intervening space. Under ordinary conditions with a small current there is a very large potential drop at the cathode and a small one at the anode. Putting lime on the cathode destroys the drop there so that Bc^2 becomes very small and the current is approximately proportional to the potential applied. With a larger current, however, the anode potential fall becomes large and Bc^2 is again large. It follows that the current increases slowly with V, as is the experimentally observed fact when V is over 300 or 400 volts.

For values of V greater than 750 volts the current begins to increase rapidly again, provided the magnetic field is zero. When the magnetic

[1] H. A. Wilson, Electrical Properties of Flames.

field acts there is a slower increase of current, not indicated in the curve of Fig. 6. This current increase is probably due to ionization by collision occurring in the regions where the ions have the highest velocity. The anode fall of potential disappears (see Figs. 4 and 5) because of the production of new ions near this region, and the new supply of negative ions being drawn to the anode produces a negative space charge. The appearance of this space charge is indicated by the change in the slope of the potential gradient curve.

From the data used in plotting Fig. 6 the velocity of the negative ions in the flame may be calculated. Townsend has derived[1] an expression for the average velocity, U_H, which an ion will acquire in the direction of an electric force, X, when a magnetic field, H, acts at right angles to X. Starting with the equations of motion

$$m\frac{d^2x}{dt^2} = Xe - He\frac{dy}{dt},$$

$$m\frac{d^2y}{dt^2} = He\frac{dx}{dt},$$

it is found that

$$U_H = \frac{U}{1 + H^2\left(\frac{e}{m}\right)^2 T^2}.$$

Here U is the velocity when the magnetic field is zero, and is equal to $X(e/m)T$, T being the mean free period of the ion. Hence

$$\frac{U}{U_H} = 1 + H^2\left(\frac{e}{m}\right)^2 T^2.$$

Let I and I_H be the currents with zero magnetic field and with field H, respectively. If the current is carried mainly by negative ions we may write

$$\frac{I}{I_H} = \frac{n_2 e_2 U}{n_2 e_2 U_H} = 1 + \frac{H^2U^2}{X^2},$$

$$\frac{I - I_H}{I_H} = \frac{H^2U^2}{X^2},$$

$$U = \frac{X}{H}\sqrt{\frac{I - I_H}{I_H}} = \frac{X}{H}\sqrt{\frac{\delta I}{I_H}}.$$

In general, the quantity under the radical depends on the value of X, so that U is not proportional to X in all cases.

In deriving the above relation it has been assumed that the electric

[1] Townsend, Electricity in Gases.

field, X, is unchanged by the magnetic field, H. In applying the equation, therefore, X was determined from the curves of Figs. 2, 3, 4, and 5 at the point where the magnetic field did not affect the potential gradient. The magnetic field had a constant value of 3,600 gauss. In the fourth column of the table below are recorded the values of U as obtained in this experiment. In the fifth column X/p is tabulated. X is measured in volts per cm. and p is taken to be 760 mm.

Applied Potential (Volts).	X (Volts).	$\frac{\delta I}{I_H}$	$U \times 10^{-5}$ (Cm. per Sec.)	$\frac{X}{p}$
75	19	0.09	1.58	0.025
149	31	0.29	4.64	0.04
300	69	0.54	14.1	0.09
450	125	0.52	24.8	0.16
600	168	0.50	33.0	0.22

Values of $\delta I/I_H$ are not calculated for larger values of the applied potential because of the presence of ionization by collision. The data in this table show that the velocity of the negative ion increases approximately in proportion to X/p except for the small values of X/p. If, in the equation $U = kX$, we consider the values of k, it is evident that k is approximately constant when X/p is large but begins to decrease when X/p grows small. It has generally been assumed heretofore that k in a flame has an approximately constant value whatever the magnitude of X/p.

Owing to the difficulty of measuring $\delta I/I_H$ accurately when X was small the velocity could not be determined for smaller values of X/p than 0.025. This limitation is unfortunate because the decrease in U and k cannot be followed out as completely as is desirable. However, from a study of the Hall effect in flames H. A. Wilson[1] has determined the velocity which the negative ion acquires under a gradient of one volt per cm., and the electric field used in his experiments was varied from 5 volts to 30 volts per cm. We are thus afforded a range in X/p of from 0.006 to 0.039. Throughout this range k was found to have a constant value of 2,450 cm. per sec., so that U would be proportional to X/p according to these experiments. The value of U varies from 0.12×10^5 cm. per sec. when $(X/p) = 0.006$, to 0.73×10^5 cm. per sec. when $X/p = 0.039$. There is a somewhat large discrepancy between Wilson's value of 0.47×10^5 when $(X/p) = 0.025$ and the value 1.58×10^5 obtained above, but this may be due to different flame conditions.

The general way, therefore, in which U varies with X/p is as follows:

[1] PHYS. REV., III., p. 375, 1914.

(1) When X/p is greater than 0.09 U is approximately proportional to X/p, (2) when X/p is between 0.09 and 0.01 (approximately) U diminishes very rapidly with X/p and (3) when X/p is below 0.01 U has become relatively small and diminishes in proportion to X/p.

Townsend[1] has shown that the velocity of the negative ions in dry air at ordinary temperatures changes in very much the same way when X/p is varied. It will therefore be important to consider the effect of a high temperature on the velocity of the ions so that a comparison can be made between Townsend's results and the results obtained for flames.

We have

$$U = X \frac{e}{m} \frac{\lambda}{V},$$

where V and λ are, respectively, the velocity of agitation and the mean free path of the ion in the gas. Introducing the constant of proportionality, A, we may express the variation of λ with the absolute temperature, θ, and the pressure, p, by the equation $\lambda = A\theta/p$. When X/p is very small we also have $mV^2 = \alpha\theta$, where α is a constant, so that for small values of X/p,

$$U = AX \frac{e}{p} \sqrt{\frac{\theta}{\alpha m}}.$$

When X/p is large it has been shown by Townsend that V becomes greater than the ordinary value given by $mV^2 = \alpha\theta$, and he has determined the factor, f, by which V^2 is increased. Hence $mV^2 = f\alpha\theta$, where f is a function of X/p. If the subscript A refers to air at 27° C. and F to the flame which has a temperature of about 2000° C., we have, when X/p is large:

$$U_A = \frac{AXe}{p} \sqrt{\frac{300}{\alpha m_A f_A}},$$

$$U_F = \frac{AXe}{p} \sqrt{\frac{2273}{\alpha m_F f_F}},$$

so that for a given value of X/p

$$\frac{U_A}{U_F} = \frac{k_A}{k_F} = \sqrt{\frac{m_F f_F \, 300}{m_A f_A \, 2{,}273}};$$

where k is defined by the equation $U = kX$. The velocity of agitation in the flame is much greater than in air at ordinary temperatures, so that the effect of the electric field on V would be comparatively unimportant.

[1] Townsend, Electricity in Gases.

We might, therefore, expect f_F to be approximately equal to 1, when the equation becomes

$$\frac{k_A}{k_F}\sqrt{\frac{2{,}273 f_A}{300}} = \sqrt{\frac{m_F}{m_A}}.$$

The following table gives some of the data obtained by Lattey[1] and by Townsend and Tizard[2] in their experiments on air at ordinary temperatures, together with the corresponding data secured in the present experiment on flame ions:[3]

Air

x/p	0.04	0.07	0.08	0.2	0.3[4]	0.5
$U_A \times 10^{-5}$	0.001	0.005	0.01	5.0	6.3[4]	9.0
k_A	3.5	9.5	15.6	3,290	2,980[4]	2,370
f_A	—	—	—	2.5	3.5[4]	5.7
$k^A\sqrt{2{,}273 f_A 300}$	—	—	—	14,400	15,200[4]	15,640

Flame.

X/p	0.007	0.025	0.04	0.09	0.16	0.22
$U_F \times 10^{-5}$	(0.12)	(0.46)	(0.74)			
		1.58	4.64	14.1	24.8	33.0
k_F	(2,450)	(2,450)	(2,450)			
		8,300	15,000	20,400	19,800	19,700

The numbers in parentheses are obtained from Wilson's experiments on the Hall effect.

If m_F is equal to m_A, k_F should be equal to $k_A\sqrt{(2{,}273 f_A/300)}$. It appears that the maximum values observed for these quantities are 20,400 and 15,640 respectively, which do not differ very much. The

[1] Proc. Roy. Soc., *A*, 84, p. 173, 1910.

[2] Proc. Roy. Soc., 88, p. 336, 1913.

[3] We may calculate the value of k_A from the relation $k = (e/m)(\lambda/V)$. If the ion is an electron we have $e/m = 1.7 \times 10^7$, $\lambda = 4.2 \times 10^{-5}$ (if the free path of an air molecule at 27° C. is 10.5×10^{-6}), and

$$V = 1.84 \times 10^5\sqrt{\frac{2 \times 300 \times 1800}{273}} = 1.1 \times 10^7$$

Hence, $k_A = 6{,}500$ cm. per sec. This agrees very well with the highest observed value of $k_A\sqrt{f_A}$ which is about 5,680. For an electron in the flame k_F may also be calculated. Taking $e/m = 1.7 \times 10^7$, $\lambda = 31.6 \times 10^{-5}$ (four times the free path of a nitrogen molecule at 2000° C.),

$$V = \frac{1.84 \times 10^5\sqrt{2 \times 2273 \times 1800}}{\sqrt{273}} = 3.2 \times 10^7,$$

we get $k_F = 16{,}800$ cm. per sec. This result agrees as well as could be expected with the largest value of k_F above.

[4] Obtained by interpolation.

mean free path of an electron in the flame is probably longer than it would be in pure air at the same temperature, a fact which would explain why the maximum value of k_F in the flame is greater than the maximum value of $k_A \sqrt{(2{,}273 f_A/300)}$.

For the smaller values of X/p the values of k_F are much greater than those of $k_A \sqrt{(2{,}273 f_A/300)}$, so that it appears that the negative ions in the flame are electrons for smaller values of X/p than in air. This conclusion was to be expected, because a high velocity of agitation would be unfavorable to the formation of aggregations of molecules about the negative ion.

The way in which the velocity of the ions varies with X/p is brought out very clearly by the curves of Fig. 7, where k_A and $2.75\sqrt{f_A} \cdot k_A$ (dotted) are plotted against X/p for air, and k_F is plotted against X/p for the flame. The name of the quantity plotted along the ordinate for

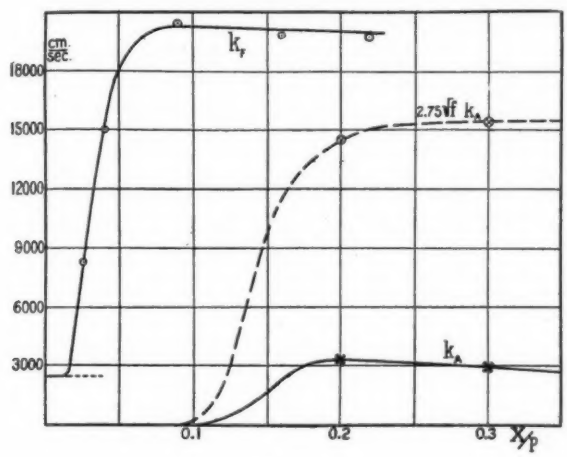

Fig. 7.

each curve is written on the curve instead of along the axis. If the mean free paths in the flame and in air at the temperature of the flame were equal, and if $f_F = 1$ in the flame, then the curves for $2.75\sqrt{f_A}\ k_A$ and for k_F would coincide if the mass of the air ion and the flame ion were the same.

The curve for k_A in air brings out[1] the following points: (1) When X/p is less than 0.01, U_A is very small and is proportional to X/p, so that k_A is constant; (2) when X/p is between 0.01 and 0.2 the value of k_A (and therefore of U_A) increases rapidly with X/p; (3) when X/p is greater

[1] The scale on which the curve is plotted is so small that (1) and (2) are not evident. A consideration of the data is necessary (see Townsend, Electricity in Gases).

than 0.2, U_A is comparatively large but does not increase in proportion to X/p, so that k_A in this case decreases as X/p increases.

Townsend explains this variation of velocity as follows: when X/p is less than 0.01 the negative ions consist of aggregations of molecules; as X/p increases a transition stage is passed through, and when the value of 0.2 is reached the ions are free electrons. The electrons do not acquire a velocity proportional to X/p when the latter is large because a large electric field increases the velocity of agitation of the ions, thus causing k_A to decrease.

In the case of the flame it is probable that very much the same phenomena occur as in the gas at ordinary temperatures. However we cannot conclude that for small values of X/p the negative flame ion never exists in an electronic state. The experiments on the Hall effect show that the negative flame ion has a much higher velocity for small values of X/p than has the ion in air. This fact indicates that its average mass is smaller. Also, even when a very small potential difference is applied to electrodes in a flame there is a greater fall of potential at the cathode than at the anode. This cathode fall of potential indicates that the negative ions have a higher velocity than the positive ions and hence a smaller mass. The conclusions to be drawn are as follows:

The electrons in the flame are, under ordinary conditions, in a state of statistical equilibrium with the molecules; that is, each flame ion spends a certain part of its life as a free electron, the rest of its life being spent as a molecular aggregate. When X/p is small this equilibrium condition is not much disturbed. As X/p changes from about 0.01 to 0.09 a transition stage is traversed, when the ion spends more and more of its time as a free electron. For values of X/p greater than 0.09 the ion is a free electron during its whole life. A rough idea of the time that an electron is associated with molecules when X/p is small may be obtained as follows:

Let k be the velocity of the free electron under unit potential gradient and k' the velocity when associated for part of the time with molecules. From the curve of Fig. 7 we then have, approximately, $k'/k = 1/8$. If t is the time during which the electron is free and t' the time it is associated we may write

$$k'(t + t') = kt + k''t'.$$

The last term on the right hand side may be neglected, because k'', the velocity of the ion when loaded with a molecule, is small when compared with k. Hence

$$\frac{k'}{k} = \frac{t}{t + t'} = \frac{1}{8},$$

$$t' = 7t.$$

The electron, therefore, is free about one eighth of the time, even when X/p is very small.

The following is a recapitulation of the main points of the experiment:

1. The potential gradient between vertical electrodes in a flame has been studied in a series of cases, the cathode being covered with a deposit of metallic oxides for the production of electrons.

2. The current-voltage curve indicates an apparent saturation current. This, however, is mainly due to the appearance of an anode fall of potential. Ionization by collision occurs for large values of the electric field.

3. By measuring the effect of a magnetic field on the conductivity of the flame the velocity of the negative ions has been determined, and k, the velocity under unit potential gradient, is found to have a maximum value of 20,400 cm. per sec.

4. The value of k for a flame is found to vary with X/p in much the same way as does k for air at ordinary temperatures. The conclusion is drawn that when X/p is above 0.09 the flame ions are free electrons, when X/p varies from 0.09 to 0.01 (approximately) the free electrons begin to associate with gas molecules, and when X/p is below 0.01 the electrons are free only about one eighth of the time. This variation of k with X/p probably explains the lack of agreement between the values of k as determined by different experimenters.

The writer is indebted to Professor H. A. Wilson for valuable suggestions and much helpful advice regarding the working out of the experiment.

THE RICE INSTITUTE,
HOUSTON, TEXAS.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NEW YORK MEETING, FEBRUARY 26, 1916.

THE Society held its eighty-second meeting in Fayerweather Hall, Columbia University, New York City, at 10:15 a.m., Vice-President Bumstead presiding. In the absence of the secretary, George B. Pegram was elected secretary pro tem.

The following program of papers was presented:

Morning Session.

On Possible Planetary Magnetic Effects. (Illustrated.) L. A. BAUER.

Note on the End Effect in the Electrostriction of Cylindrical Condensers. EDWIN C. KEMBLE.

A New Method and Apparatus for Enlarging Photographs without the Aid of a Lens. ALFRED J. LOTKA.

The X-Ray Spectrum of Tungsten. ARTHUR H. COMPTON.

A New Mercury Vapor Pump. HORATIO B. WILLIAMS.

Distribution of Velocities of the Electrons in Vacuum Tubes. K. T. COMPTON.

Reversed Sounds. JOHN B. TAYLOR.

Theory of Ionization of Gases by Elastic and Inelastic Impact. K. T. COMPTON.

Fundamental Ideas in Photography. F. M. STEADMAN.

Osmotic Pressure and Temperature. W. F. MAGIE.

Afternoon Session.

Behavior of a Stable Aeroplane in Longitudinal Gusts. E. B. WILSON.

Some Afterglow Phenomena in Nitrogen and Argon. IRVING LANGMUIR AND C. U. FERGUSON.

Rate of Evaporation of Small Spheres in Air. IRVING LANGMUIR.

Theory of Crystal Structure. A. C. CREHORE.

The question of endorsing a bill now before Congress proposing to make the Centigrade thermometer scale the official scale in the publications of the various government bureaus and to abolish by 1920 the use of the Fahrenheit scale in government publications, was discussed at some length and it was

finally decided to postpone action upon it until the Washington meeting in April.

G. B. PEGRAM, *Secretary pro tem.*,
A. D. COLE, *Secretary.*

MINUTES OF THE WASHINGTON MEETING, APRIL 21 AND 22, 1916.

THE Eighty-third Regular Meeting of the Physical Society was held at the National Bureau of Standards, Washington, April 21 and 22, 1916. The following papers were presented:

Friday, April 21, 9:30 A.M.

Convection Currents and Calorimeter Design. WALTER P. WHITE.

The Application of the Moving Coil Galvanometer to the Measurement of Time. PAUL E. KLOPSTEG. Read by A. Zeleny.

On the Influences Contributing to the Variation of Contact Electromotive Force with Time. OTTO STUHLMANN, JR.

The E. M. F. Generated by the Rotation of a Cylindrical Magnet About its Axis. G. F. HULL AND A. B. MESERVY.

A High Vacuum Mercury Vapor Pump of Extreme Speed. IRVING LANGMUIR.

The Effect of a Magnetic Field Upon Solid Solutions of Iron and Carbon. F. L. BISHOP. (By title.)

The Combination of a Series and a Parallel Resistance for Reducing the Voltage Sensitivity of a Galvanometer. F. WENNER.

An Alternating Current Method for the Measurement of Mutual Inductance. HARVEY L. CURTIS.

The Resistance of Thin Films Produced by Evaporation. S. DUSHMAN.

On a Chemically Active Modification of Hydrogen Produced by Alpha-Rays. WILLIAM DUANE AND G. L. WENDT.

The Volume Effect in the Silver Voltameter. E. B. ROSA AND G. W. VINAL.

The Law of Absorption of X-Rays at High Frequencies. ALBERT W. HULL.

Stoletow's Constant: Its Theoretical Derivation and Application to Measurements of Electronic Free Paths. K. T. COMPTON. (By title.)

The Calculation of Planck's Constant C_2. J. H. DELLINGER.

Note on the Nature of the Absorbers of Characteristic X-Rays. R. A. MILLIKAN AND E. C. WATSON.

The Low Potential Discharge Spectrum of Mercury Vapor in Relation to Ionization Potentials. JOHN T. TATE.

2.30 P.M.

Photo-currents at Different Wave-lengths as a Function of Incident Energy. WILMER SOUDER.

On the Conduction of Electricity through an Ionized Gas, more particularly in its Relation to Bronson Resistances. W. F. G. SWANN AND S. J. MAUCHLY.

An Experimental Determination of the Mean Free Paths of Electrons in Rarified Gases. K. T. COMPTON AND J. M. BENADE. (By title.)

Some Properties of Mercury Spheres Important in Electronic Charge Measurements. L. W. MCKEEHAN.

On the Mobilities of Gas Ions in High Electric Fields. L. B. LOEB.

Wave-length Energy Distribution in the Continuous X-Ray Spectrum. BERGEN DAVIS.

On Spectroscopic Resolving Power. C. M. SPARROW.

The Planetary System of Convection as Observed by Means of Sounding Balloons. WM. R. BLAIR.

The Vertical Distribution of Air Temperature and of Air Pressure in Middle Latitudes. WM. R. BLAIR.

Saturday, April 22, 9:30 A.M.

The Constants of Radiation of a Black Body. W. W. COBLENTZ.

On the Use of the Thermopile in Photographic Photometry, with Special Reference to Spectroscopy and the Determination of Stellar Magnitudes. H. T. STETSON.

The Light Absorption Coefficient of Solutions. E. O. HULBURT, J. F. HUTCHINSON AND H. C. JONES.

The Candle-power of the Black Body and the Mechanical Equivalent of Light. E. P. HYDE, F. E. CADY AND W. E. FORSYTHE.

The Under-water Spark as an Ultraviolet Source. H. E. HOWE.

The Rotating Sector in Photographic Photometry. H. E. HOWE.

The Law of Reflection of Gas Molecules from Liquid and Solid Surfaces. R. A. MILLIKAN.

The Efficiency of Thermostats. WALTER P. WHITE.

Rates of Evaporation and Vapor Pressures of Iron, Nickel, Silver and Copper. IRVING LANGMUIR AND G. M. J. MACKAY.

A Synchrono-phosphoroscope. EDWARD L. NICHOLS AND H. L. HOWES. (By title.)

Note on the Action of Calcium in a Discharge Tube. HAROLD B. C. ALLISON.

A Misconception of the Criterion for Gray-body Radiation. PAUL D. FOOTE AND C. O. FAIRCHILD.

The Thermal Expansion of Tungsten at Incandescent Temperatures. A. G. WORTHING.

American Nickel Steels of Low Thermal Expansivity. A. W. GRAY, D. H. SWEET AND L. W. SCHAD.

Some Effects of Magnetization on the Length of a 36 Per Cent. Nickel Steel. A. W. GRAY, D. H. SWEET AND L. W. SCHAD.

The Intensity of X-Ray Spectra, and the Distribution of the Electrons in Atoms. ARTHUR H. COMPTON.

Experiments on the Joule-Thomson Effect in Air at Moderate Temperatures and Pressures. L. G. HOXTON.

An Automatic Pressure Regulator. L. G. HOXTON.

An Investigation of the Laws of Plastic Flow. E. C. BINGHAM.

Dynamics of Electron Fields. G. B. PEGRAM.

Note on a Method of Measuring Thermocouple E.M.F.'s. E. F. NORTHRUP.

During the Saturday session several items of business were presented by the Council to the Society. First they presented a report from a committee on the extension of the influence of the Society among teachers of physics, consisting of G. W. Stewart, A. G. Webster and W. S. Franklin, and moved the adoption of the following recommendations of this report:

"1. That the Council be instructed to formulate and to recommend such amendments to the By-laws as are necessary to establish a *student membership*."

"2. That the Council be authorized to establish, if feasible, a special PHYSICAL REVIEW subscription rate for individuals who are members of societies that are interested in the teaching of physics, the subscriptions being made through the secretaries of such organizations."

"3. That the Council be authorized to appoint a representative of the Society who shall prepare for each issue of School Science and Mathematics a record of some of the most interesting recent achievements in physics, to make such an appointment annually and to discontinue such representation at its own discretion."—Motion carried.

The second item recommended by the council for adoption by the Society was the following:

"The American Physical Society shares the desire of scientific men in general for international and world-wide uniformity in units of measurement of all kinds, and with this object in view it favors the introduction of the Centigrade scale of temperature, as a standard in the publications of the United States government.

"It must be recognized that considerable initial expense must be incurred by the U. S. Weather Bureau in changing its apparatus to conform to the proposed act. Furthermore, on account of the more open scale of the Centigrade system that Bureau will be subject to a continued increased cost of publication, owing to the necessity of printing the first decimal place in order to maintain the present accuracy. The use of negative temperatures and minus signs entails greater liability to errors, and more clerical labor would be required in checking the accuracy of the reports of coöperative observers of the Weather Bureau, and in computing monthly and other mean temperatures.

"Notwithstanding the foregoing, the American Physical Society is in favor of legislation to make the Centigrade scale of temperatures the standard in publications of the United States government, and funds should be made available by Congress to accomplish the desired result.

"The Society favors Bill H. R. 528, 'To Discontinue the Use of the Fahrenheit Thermometer Scale in Government Publications,' but recommends that it be amended by the addition of the following:

"*Sec. 4.* When in the publication of tables containing several meteorological and climatic elements, the use of data in Centigrade temperatures leads to manifest incongruities, the chief of the Weather Bureau is directed to publish related data in such units as are necessary to make the tables homogeneous and to secure international uniformity as far as practicable.

"*Sec. 5.* Nothing in this act shall prevent the use of the absolute Centigrade scale of temperature in publications of the government."

The motion was adopted without a dissenting vote.

The Committee on Technical Physics (F. K. Richtmyer, Chairman) also made an extended report of progress, neostyle copies of which were distributed. This report is unfortunately too long to be included here; it suggested seven lines of activity in which the Society might engage with the object of increasing the usefulness to the physicists of the country of the physical investigation now being conducted in the laboratories of industrial organizations. Seven further suggestions were made of ways in which individual members might help to secure better coöperation between university and industrial laboratories.

A vote of thanks was extended to the Washington members of the Society and to the Bureau of Standards for numerous courtesies extended to visiting members. It was announced that the meeting of the Society next October will probably be held at the Nela Research Laboratory of the General Electric Company, Cleveland, O.

A considerable number of members of the Physical Society took advantage of the special invitations extended to them to attend the open sessions of the National Academy of Sciences, April 17, 18, and 19, and the address before the Philosophical Society of Washington by Professor R. A. Millikan the evening of April 20.

On both days of the meeting visiting members of the Society were guests of the Washington members of the Society at luncheon at the Bureau of Standards. On Friday evening a subscription dinner at the Cosmos Club was participated in by about fifty. The attendance at each of the three sessions for the reading of papers was large—probably 125 to 150 at each session.

A. D. COLE,
Secretary.

In accordance with a vote of the Council of the American Physical Society, the ARTICLES OF INCORPORATION of the Society, adopted Dec. 30, 1914, are here recorded:

Articles of Incorporation of The American Physical Society.

Know All Men by These Presents, that we, all citizens of the United States, whose names are hereto undersigned, a majority of whom are residents of the District of Columbia, being desirous to form a society and body corporate under the laws of the said District, and in particular under sub-chapter III of Chapter XVIII of the Code of Law in force in the said District, do hereby declare and certify as follows, that is to say:

First. The name of the Society hereby formed shall be The American Physical Society.

Second. The term of existence of the said society shall be perpetual.

Third. The said society shall have no capital stock.

Fourth. The object and purpose of the said society is, and shall be, to promote the advancement and diffusion of the knowledge of physics, to publish a periodical and other publications for that purpose, and to do such other things as may be conducive to the said purpose.

Fifth. The business affairs and concerns of the said society shall be, during the first year of the existence, managed and directed by a board of trustees to be called the council and consisting of twelve persons, the first members of which council, who shall act as such until the membership may be changed by an election held in accordance with the constitution and by-laws of the society, shall be:

Ernest Merritt, Ithaca, New York;
Karl E. Guthe, Ann Arbor, Michigan;
Alfred D. Cole, Columbus, Ohio;
Joseph S. Ames, Baltimore, Maryland;
Robert A. Millikan, Chicago, Illinois;
John Zeleny, Minneapolis, Minnesota;
Henry A. Bumstead, New Haven, Connecticut;
Theodore Lyman, Cambridge, Massachusetts;
Howard T. Barnes, Montreal, Canada;
William J. Humphreys, Washington, D. C.;
Dayton C. Miller, Cleveland, Ohio;
George K. Burgess, Washington, D. C.

The said society reserves the right and power to amend, alter, replace and add to any and all of the provisions contained in this certificate in such manner and to such extent as may be authorized by law.

In witness whereof, we have affixed our signatures and our seals this the fourth day of November in the year 1914.

ARTHUR L. DAY, (SEAL)
W. J. HUMPHREYS, (SEAL)
S. W. STRATTON, (SEAL)
GEORGE K. BURGESS, (SEAL)

(Sworn statements before Notary Public and Deputy and Acting Recorder of Deeds accompany the original.)

CONVECTION CURRENTS AND CALORIMETER DESIGN.[1]

BY WALTER P. WHITE.

IN calorimeters surrounded by air the cooling factor ("thermal leakage" factor) varies with the temperature difference between the calorimeter and the inclosing jacket. To avoid this difficulty it is necessary either to work with small differences or else to make certain corrections which, while not especially troublesome, are preferably to be avoided. The variation can be made negligible by a suitable proportioning of the calorimeter and its surrounding jacket. The heat transfer by conduction through the air and by radiation is, under ordinary conditions, practically proportional to temperature difference; these sources of thermal leakage contribute practically nothing to the variation in the leakage factor. The whole trouble comes from the convection currents, which, in thin air layers, transfer heat at a rate which is practically proportional to the square of the temperature difference. By making the space between calorimeter and jacket narrow enough the effect of these currents can be made negligible and the desired constancy in the thermal leakage factor can be sufficiently approximated. This construction, however, while diminishing the convection currents, increases conduction, and therefore should not be carried too far.

In order to get data for determining the optimum distance between calorimeter and jacket the convection through vertical and horizontal spaces was investigated. The apparatus consisted of two silvered plates of copper placed back to back within a rectangular copper box. The box was heated in an oil bath and the rise in temperature of the copper plates served as a measure of the heat flowing to them through the air spaces from the walls of the box. The temperatures were measured by thermoelements soldered to the copper surfaces. 0.1 microvolt, the smallest quantity read, corresponded to 0.0025°, and the final agreement was frequently, though not always, as good as this. Conduction and radiation were determined for a small temperature difference where convection was practically absent, and the heat flow due to these was then subtracted to get the convection in the determinations where the temperature difference was greater. The advantages of this method are: its simplicity and relative rapidity, the ability to work accurately for very small temperature differences, and the ability to work over a rather wide temperature range. The absence of guard rings is perhaps an advantage, since conduction and radiation entered only as a correction term, and the convection currents deliver their heat at the very edge of the plate.

Vertical flow was mainly studied. For distances profitable in calorimetry stream line flow prevails. The fundamental fact, therefore, about the convection currents is that, flowing parallel to the solid surfaces, they carry no heat to and from these except when they first strike or when they leave. It follows that for surfaces not too low (1) convective transfer is independent of

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

height, that is, the transfer per square cm. is inversely proportional to height; (2) the stream velocity is proportional to temperature difference; (3) the velocity for infinite plates is theoretically proportional to the third power of the distance separating them. Practically, owing to end effects, etc., it proved more nearly proportional to the fourth power, and its behavior in this regard would depend on the special conditions of the particular apparatus used. The variation from Newton's law, therefore, is inversely proportional to height, proportional to temperature difference and approximately proportional to the fourth power of the width of the air gap.

The convection between horizontal surfaces is not so simple, but its magnitude was found to be for a diameter of 8 cm. approximately double that between the same surfaces put vertically. Since convection takes place from but one of the horizontal surfaces of the calorimeter at a time, these horizontal surfaces may for all practical purposes be calculated as if they were vertical and 8 cm. high.

On account of the rapid increase of convection with distance between the plates the profitable limits for that distance are narrow. As far as the lateral surface is concerned, for a calorimeter 16 cm. high and an air gap of 12 mm. the variation from constancy in the thermal leakage factor is approximately 6 per cent. when the temperature interval is raised to 10°, and this variation would be doubled by a further increase of 3 mm., which would only decrease the total cooling rate about 20 per cent.

With a very large calorimeter, however, in which the temperature difference is likely to be considerably less, the air gap may often profitably be increased in proportion to the other linear dimensions, and such an increase in the air gap makes the relative performance of a large calorimeter far more advantageous than it has generally been considered to be, since the discussion of such performance has usually and unconsciously assumed an invariable width of gap. It further follows that the lag effect due to the convection currents, discussed by several writers on calorimetry, is altogether negligible. This conclusion confirms, but also extends, one announced several years ago. The lag effect due to the air itself can be calculated from the specific heat and thermal conductivity. I must also retract a statement I once made to the effect that there is no disadvantage in having a calorimeter chamber much larger than the calorimeter. The disadvantage, however, since it concerns only the variation from Newton's law, is not serious.

Geophysical Laboratory,
Carnegie Institution of Washington,
Washington, D. C.
March 28, 1916.

SOME EFFECTS OF MAGNETIZATION ON THE LENGTH OF A 35.25 PER CENT. NICKEL STEEL.[1]

BY A. W. GRAY, D. H. SWEET, AND L. W. SCHAD.

WHILE investigating the thermal expansivity of a 35.25 per cent. nickel steel, interesting and valuable results were obtained by studying the effects of magnetic treatments at temperatures ranging from $-75°$ C. to over 300° C. Combined thermal and magnetic treatment was made to effect a permanent increase in the length at room temperatures that amounted to more than 55 microns per meter. This length change is in the same direction as that produced by Guillaume's aging process but is far more rapid. Guillaume found a hot-forged bar left to itself at room temperature to elongate 38 microns per meter in the course of 12.4 years. Artificial aging by gradual cooling from 150° C. for nearly four months produced in a similar bar an elongation of 47 microns per meter, which increased to 53 microns per meter after remaining at room temperature 4.6 years longer. The greatest aging effect recorded by Guillaume seems to be 66 microns per meter, which was produced by the above heat treatment followed by over thirteen years rest at room temperatures. Since considerable elongation had already taken place before we began to observe the 55 microns mentioned above, it would appear that our combined thermal and magnetic treatment is more rapid and more effective than the aging process of Guillaume. In addition to producing this elongation at room temperatures, the magnetic treatment raises the expansion curve as a whole at all temperatures below that at which ferromagnetic properties disappear on heating. Also, the length assumed at any temperature immediately after magnetic treatment is very nearly independent of whether this temperature is reached by warming or by cooling.

Measurements of linear magnetostriction at temperatures distributed between $-75°$ C. and 300° C. have confirmed and extended the observations of Nagaoka, Honda, and Shimizu, which appear to be the only magnetostriction measurements so far made on nickel steels. Complete magnetostriction cycles showing hysteresis, similar to those first observed by Nagaoka in the case of iron but hitherto not investigated in the case of nickel steels, have been taken at a series of temperatures up to that at which the magnetostriction vanishes, and magnetic elongation exceeding 45 microns per meter have been measured. This appears to be by far the largest longitudinal magnetostriction effect on record for any substance, exceeding by 45 per cent. the greatest noted by Honda and Shimizu with a nickel steel cooled in liquid air, and by 18 per cent. the maximum contraction recorded by Nagaoka and Honda for nickel.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
April 5, 1916.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

AMERICAN NICKEL STEELS OF LOW THERMAL EXPANSIVITY.[1]

BY A. W. GRAY, D. H. SWEET, AND L. W. SCHAD.

IN order to assist American manufacturers in producing nickel steels of specified thermal expansivity combined with other desirable properties, the Bureau of Standards has coöperated with the Midvale Steel Company in an investigation of alloys containing about 36 per cent. nickel. The expansivity has been studied at many temperatures extending from about − 150° C. to about 665° C., and data have been obtained that show effects of mechanical, thermal, and magnetic treatments, as well as of slight variations in the constituents. The investigation has already resulted in the production of specimens which, as far as small expansivity is concerned, compare favorably with good French invar; but sufficient time has not yet elapsed to show how stable they can be made as regards length.

In conformity with the experience of Guillaume it has been found that cold-drawing and quenching lower somewhat the expansivity at room temperatures, while annealing raises it. Also, the curve representing the expansion on heating was found to lie above that representing the contraction on return to room temperature, except in the case of a cold-drawn specimen, in which the relation was reversed. At any temperature, however, above that at which the alloy assumes ferromagnetic properties on cooling, the length was always found to be the same whether the temperature in question was reached by warming or by cooling. At very low temperatures the expansion curve rises slowly with slight convexity upwards. At ordinary temperatures this convexity flattens out as the curve passes through a region of inflection that has a very gentle slope. In the neighborhood of 200° C. the direction of curvature is reversed, giving place to a strong concavity upwards as ferromagnetism rapidly disappears. At higher temperatures the concavity is slight but the slope is steep.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
April 5, 1916.

THE E.M.F. GENERATED BY THE ROTATION OF A CYLINDRICAL MAGNET ABOUT ITS AXIS.[1]

BY G. F. HULL AND A. B. MESERVEY.

AN electromagnet with poles in the form of hemispherical shells is made to rotate about its axis of symmetry. A ring of brass or iron is held at rest between the poles. Brushes rotating with the magnetic shells make contact with the inner and outer edges of the ring and are connected to insulated metal rings on the axis. When the magnet rotates, an E.M.F. is picked up

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by sliding contact on these metal rings. The amount is equal to that which is computed from the product Nn when N is the total magnetic flux through the fixed ring and n is the number of revolutions per second. If we adhere to the idea of magnetic lines of force, quantitatively the result is in agreement with the view that the lines of force rotate with the magnet and sweeping across a fixed conductor produce in it an E.M.F. equal to Hlv. The experiment is being continued.

DARTMOUTH COLLEGE,
HANOVER, N. H.

THE LOW POTENTIAL DISCHARGE SPECTRUM OF MERCURY VAPOR IN RELATION TO IONIZATION POTENTIALS.[1]

BY JOHN T. TATE.

IT has been shown by Franck and Hertz that collisions between electrons accelerated in an electric field and atoms of mercury vapor are elastic for velocities of the electrons less than 4.9 volts. At this velocity the electrons are capable of giving up all their energy at collision. They were also able to show that part, at least, of this energy was emitted by the vapor as monochromatic radiation of wave-length 253.67 $\mu\mu$—the frequency to be expected if we put

$$h\nu = eV.$$

Incidentally the writer has repeated the measurements of Franck and Hertz very carefully and has obtained a value of $4.90 \pm .03$ volts for the critical velocity. Indications of secondary maxima in the current-potential curves were found showing the possibility of other types of collision at higher potentials.

Franck and Hertz interpret their results to mean that an atom of mercury vapor is ionized by collision with an electron possessing a velocity of 4.9 volts. It should be noted however that their experimental evidence does not definitely prove this. All that is clearly demonstrated is that the collisions become inelastic at that point. It is readily conceivable, for instance, that the energy lost by the colliding electron merely goes over into energy of agitation of the electrons bound in the atom and not necessarily into completely separating one or more electrons from the atom.

However that may be, it is certain that a secondary critical velocity of the electrons is to be expected at which the energy they give up is emitted as radiation of many frequencies corresponding to the many-lined spectrum of mercury. The present paper deals with an attempt to fix the value of this secondary critical velocity and to demonstrate whether or not ionization takes place at this point. McLennan and Henderson, investigating the character of the single- and many-lined spectra of various elements state that the many-lined

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

spectrum of mercury appears when the applied potential is greater than 12.5 volts. This value however can only be a rough approximation.

The apparatus used in this investigation consisted of a discharge tube into which a mercury barometer column projected giving a continuous supply of mercury vapor when heated by a hot cathode situated about 2 cm. above the surface of the mercury. A platinum net screening the mercury surface served as anode. To further concentrate the field a second platinum net was introduced between hot cathode and anode. The space between nets was focused on the slit of a quartz spectrograph in such a way that the lines of the spectrum were limited in length by the nets. The initial velocity of the electrons was obtained in the usual way.

It was found that with velocities corresponding to total effective potentials of more than about 5 volts and less than 10 volts the spectrum consisted of a single line of wave-length 253.67 $\mu\mu$. At about 10.3 volts the many-lined spectrum of mercury began to appear in the region of the anode and with increasing potentials the lines gradually increased in length. To determine this critical potential more accurately, and at the same time to demonstrate whether or not ionization took place there, current-potential curves were taken. A very sharp break in the curves was found at 10.0 volts. No break in the curves was detected in the region of 4.9 volts.

The results of the investigation show (1) that a marked ionization occurs in mercury vapor when the velocity of the colliding electrons reaches a critical value of 10.0 volts (possible error about .3 volt). This is very nearly the value (10.2 volts) to be expected on the basis of Bohr's theory of the atom as McLennan has pointed out. (2) That the energy lost by the electrons at these collisions is radiated out as the many-lined spectrum of mercury. (3) That although ionization of mercury vapor at 4.9 volts is not definitely disproved it is certainly much less complete than the ionization taking place at 10.0 volts.

THE BRACE LABORATORY OF PHYSICS,
THE UNIVERSITY OF NEBRASKA.

THE CALCULATION OF PLANCK'S CONSTANT C_2.[1]

BY J. H. DELLINGER.

THIS important constant can be calculated directly and simply from the observed ratio of the energies at any two wave-lengths and temperatures. A rigorous solution of Planck's radiation equation is readily obtained, and the following approximate expression is sufficiently exact for most cases.

$$C_2 = \frac{\lambda_1\lambda_2\theta}{\lambda_2 - \lambda_1}\left[\log\frac{J_2}{J_1} + 5\log\frac{\lambda_2}{\lambda_1} - \epsilon^{-C_2/\lambda_2\theta}\right]$$

An approximate value of C_2 always suffices for the last term. The various other relations which have been used for obtaining C_2 from energy data are

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

deducible from the general relation as special cases. The equation for isothermal curves is of particular interest. This general method of solution is very much superior to the method of equal ordinates. No curve has to be drawn, and the calculations are not limited to particular pairs of points. The method is more powerful in determining whether an observed curve fits the Planck equation. In fact, curves which give normal values for C_2 by the method of equal ordinates are found to give very high values when calculations are made by this method for two points both on the same side of the maximum.

BUREAU OF STANDARDS,
WASHINGTON.

NOTE ON THE ACTION OF CALCIUM IN A DISCHARGE TUBE.[1]

BY H. B. C. ALLISON.

DURING the course of some experiments on gaseous conduction, a numbef of Plücker tubes containing various gases were made. The action or one of these tubes was considered worthy of note.

The tube was of the usual type, having a capillary connecting the two electrode chambers, and was about eight inches in length. The electrodes were aluminum rods mounted on platinum wires. In one of the electrode chambers some calcium chips were placed before the electrode was sealed into the tube.

The usual lamp exhaust was given, and the tube filled with argon of at least 99.8 per cent. purity. After washing out the tube twice it was sealed off at 2 mm. pressure and connected with a high voltage transformer. A spectroscope having been put in position, the spectrum was observed to be a combination of argon lines and nitrogen bands, with four prominent hydrogen lines.

As the tube continued to run the nitrogen disappeared first, and then the hydrogen, until only the pure argon spectrum remained. During this time the calcium chips were in the path of the discharge, but not in contact with the electrode.

The tube was then inverted, and shortly the hydrogen lines appeared as brilliantly as at first. The calcium was then in contact with the electrode. Upon returning to the original position the clean up of the hydrogen again took place. It was possible to repeat this reversal a large number of times, but as long as any calcium remained in contact with the electrode, hydrogen was always present in the spectrum.

This is undoubtedly due to the formation of calcium hydride in the first position, and its subsequent decomposition when in contact with the electrode.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

A CHEMICALLY ACTIVE MODIFICATION OF HYDROGEN PRODUCED BY ALPHA-RAYS.[1]

BY WILLIAM DUANE AND G. L. WENDT.

IT is well known that the rays from radioactive substances produce chemical reactions. They decompose water, whether the water is in the solid, liquid or gaseous phase; they generate ozone when they pass through oxygen, etc.

We performed the experiments described in this paper for the purpose of determining whether or not the rays cause a chemical change in pure hydrogen.

The hydrogen was prepared by the electrolysis of a ten per cent. solution of redistilled hydrochloric acid contained in a sealed two-liter Wolff bottle. A sheet of platinized platinum formed the cathode and bars of purest commercial zinc immersed in redistilled mercury, the anode. Such a generator furnishes its own current. It evolves hydrogen of exceptional purity, since the chlorine is absorbed by the zinc amalgam to form zinc chloride and never forms the gaseous phase. The hydrogen was purified by passing it through three Emmerling towers filled with lumps of potassium hydroxide which had been fused in an iron dish with potassium permanganate to remove organic material. Thence it flowed through a hard glass tube filled with platinized asbestos and kept at a red heat. Passing then through three additional KOH towers it was finally dried in a long tube of phosphorus pentaoxide. This process completely freed it from acid spray and vapors, from CO_2, Cl_2, from oxygen and from water. Any arsine which may have been formed from an impurity of arsenic in the zinc of the generator would have been decomposed by the red-hot tube, but long experience with this type of hydrogen generator has shown that no arsine is developed even with very impure zinc. The hydrogen could have been contaminated then only with nitrogen and the rare gases coming from air dissolved in the acid of the generator. These were obviated by exhausting the whole system by means of a water aspirator to very close to the vapor pressure of the solution. Hydrogen was then generated until the pressure reached atmospheric value. The exhaustion was then repeated three times. Following this treatment the entire system was swept out by its own hydrogen for 40 hours before any hydrogen was used. Contamination from dissolved air at the end of this time was out of the question. No impurities from stop-cock grease were possible because only mercury-sealed U-tubes were used in place of stop-cocks and all stoppers were sealed with mercury.

When a five c.c. bulb of this hydrogen was radiated from the interior with the alpha radiation from 35 millicuries of radium emanation in an alpha-ray tube a decrease in volume resulted. We measured the decrease by the change in level of the mercury in two branches of a U-tube, one connected to the radiated bulb, the other to a similar bulb filled with the same hydrogen and

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

mounted with the first bulb in a water-bath to equalize their temperatures. We must use caution in interpreting this change of volume. It may be due to an agglomeration of hydrogen atoms and molecules, and it may also be the result of some effect on the surface of the mercury.

To investigate the chemical properties of the gas a stream of hydrogen was passed through a somewhat larger bulb and during its passage through the bulb was exposed as before to the radiations from radium emanation. From this ionization chamber the hydrogen was passed through a glass tube containing flowers of sulfur and thence over a strip of filter paper moistened with acetate of lead. In the course of five or six hours the paper gave a decided record of the formation of hydrogen sulfide, though in quantities less than a milligram. When the emanation was withdrawn the stream of hydrogen passed for days over the sulfur without producing the slightest trace of sulfide. This form of hydrogen, then, is much more active chemically than is the ordinary form. Furthermore it is unstable, as would be expected, because the amount of sulfide detected could be varied at will by regulating the speed of the stream of hydrogen. Reducing to one half the time elapsing between the exposure to the rays and the contact with the sulfur considerably increased the amount of H_2S formed. If more than a minute was allowed to elapse between the formation of the new gas and its action on the sulfur very little sulfide could be detected.

Before the gas reached the sulfur it passed through a plug of clean glass wool. This would indicate that the chemical effect is not due to charged ions that had not yet recombined. To make certain that ions are not concerned, the hydrogen after exposure to the ionizing rays was passed through electric fields up to of 2,000 volts per centimeter in such a way that the gas remained in each field for at least half a minute. This effectually removed all ions, since the mobility of the hydrogen ion at atmospheric pressure is 9 cm. per sec. per volt per cm. The sulfide test was unaffected by this treatment; and the activity is therefore due to electrically neutral molecules.

To dispose further of the objection that the chemical effect may be due to an impurity in the hydrogen which becomes active under the influence of the alpha rays—the rays effect the activation in any case—hydrogen prepared by the electrolysis of a weak solution of potassium hydroxide containing barium hydroxide between platinum electrodes was used, as well as hydrogen bought in the market as electrolytic. Both were purified as was the first hydrogen. Both gave exactly the same effects. In many experiments the hydrogen was passed through a long spiral immersed in liquid air before entering the ionizing bulb, and this also had no effect in modifying the observations.

The reaction with sulfur is not the only one possible, although it was used preferably as a test because the reaction with lead acetate is so singly a test for sulfide. If any oxygen, for instance, had passed with the hydrogen and had become ozonized it would have attacked the sulfur to form oxides, but these when they reached the lead acetate paper would in every case have turned it

white, whether through the formation of sulphites or sulfates. There is no misinterpreting the black color produced with the lead salt.

The active hydrogen also attacks phosphorus, as evinced by the blackening of silver nitrate paper by the phosphine formed. Similarly arsine is formed by its action on powdered gray arsenic, the test here being the Gutzeit test, blackening of mercuric chloride paper. It was thought that perhaps the unknown hydride of bismuth could be prepared in the dry way by this hydrogen, but no evidence for it was obtained, using Marsh's test by decomposition in a warmed glass tube. It is conceivable but hardly probable that BiH_3 was formed but not decomposed at the temperature used in the test. When passed over the surface of pure mercury the active hydrogen forms a lustrous yellow scum which may be the hydride, though not enough was obtained to make an analysis.

Neutral potassium permanganate solution is at once reduced by this new hydrogen to manganese dioxide. It was not possible, however, to reduce the organic dyestuffs indigo carmine or methyl violet by bubbling the gas through their water solutions.

Passing the hydrogen through a spiral immersed in liquid air after the exposure to ionization but before contact with sulfur destroyed its ability to attack the sulfur, although exposure to liquid air before ionization in no way affected the test. It may be that the active gas is not truly liquefied but only adsorbed by the glass walls at the low temperature. Experiments to test the effect of red heat on the gas were not successful. The ionized hydrogen was passed over a spiral of fine platinum wire wound on a quartz rod and heated to about 600°, then through a cooling coil and over the sulfur, but it has not yet been possible to heat the gas to 600° and cool it down again within a minute to a temperature at which ordinary hydrogen does not attack sulfur. Even at 200° ordinary hydrogen attacks sulfur appreciably.

The atomic mechanism of this phenomenon is not yet clear. Atoms of hydrogen may be fired into other atoms or molecules by the direct bombardment of the alpha particle or the new form may be the result of the breaking down of a large aggregate of molecules which had collected on a charged molecular fragment. It is conceivable that such an aggregate, formed by the ionization, while disintegrating may become chemically more active than in the normal state. No evidence is at hand to decide this point. An attempt was made to place so high an electrostatic field across the ionizing field that ions would be removed at once without having opportunity to produce large aggregates A field of 2,000 volts per cm. however, placed directly across the region of greatest ionization had no appreciable effect. This may indicate that the primary bombardment is responsible for the production of the active state.

Whether or not the rays produce the compound H_3 remains undetermined.

HARVARD UNIVERSITY.

AN ALTERNATING CURRENT METHOD OF MEASURING MUTUAL INDUCTANCE.[1]

BY HARVEY L. CURTIS.

THE mutual inductance between two coils is defined as the electromotive force induced in one when a current in the other changes at the rate of one ampere per second. If an alternating current $i_1 = I_1 \sin \omega t$ flows in one coil then the induced electromotive force e_2 is given by the equation

$$e_2 = I_1 M \omega \cos \omega t \qquad (1)$$

provided that no current due either to leakage or capacity flows between the two coils or between the turns of either coil. If such currents do flow, then the secondary electromotive force e_2, measured at the terminals of the secondary with the secondary circuit open, may be represented by the equation

$$e_2 = I_1 M_e \omega \cos \omega t + I_1 \sigma \sin \omega t \qquad (2)$$

where M_e is the effective mutual inductance and σ a small factor having the dimensions of a resistance.

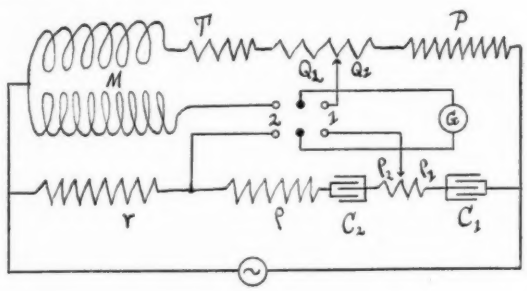

Fig. 1.

The open-circuit values of σ and M_e may be measured by means of the set-up of Fig. 1. The detecting instrument G is a vibration galvanometer or telephone, depending upon the frequency of the alternating current used. The inductances of the resistances r and P should be small and known. The resistances Q_1 and Q_2 are so designed that in varying Q_1, the sum $Q_1 + Q_2$ is always the same. This same condition also applies to ρ_1 and ρ_2; viz., $\rho_1 + \rho_2 =$ a constant.

With the galvanometer switch in position 2, T and ρ are alternately adjusted until no current flows through the galvanometer. The balance can only be obtained provided the self inductance in T is greater than M, so that it may be necessary to add inductance to this arm. The switch is then changed to position 1, and Q_1 and ρ_1 adjusted for zero current in the galvanometer circuit. Then, neglecting small correction terms,

$$M_e = C_1 r (P + Q_1)$$

and

$$\sigma = \omega^2 C_1 \rho_1 M_e.$$

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

If σ is very small, it may be impossible to make ρ_1 sufficiently small to obtain a balance. In such a case, a small inductance l is introduced in P. Then $\sigma = \omega^2 C_1(M_e\rho_1 - rl)$ and M_e is unchanged.

One of the advantages of the method is that the resistances r, P and Q_1 upon which the value of M_e depend can be so constructed that the change of resistance with frequency and with temperature is a minimum. This in a measure compensates for the disadvantage of having to make four independent adjustments.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE COEFFICIENT OF TOTAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE.[1]

BY W. W. COBLENTZ AND W. B. EMERSON.

A RECENT determination of the coefficient, or so called Stefan-Boltzmann constant, of total radiation of a black body gave a value of $\sigma = 5.72 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$. The radiometer used in measuring the radiant energy in absolute value consisted of a bismuth-silver thermopile at a short distance in front of which was placed a thin strip of metal of manganin or platinum, from 0.008 to 0.012 mm. in thickness, from 2 to 8 mm. in width and 30 mm. in length. Potential terminals of very thin platinum wire were attached at a distance of 2 to 3 mm. from the ends. This thin strip of metal, which was blackened electrolytically with platinum black, or painted with lamp black and then smoked, functioned (1) as a receiver for absorbing radiant energy, (2) as a source of radiation which can be produced by heating the strip electrically and (3) as a standard of radiation to test the galvanometer sensitivity, by heating the strip electrically by a standard current.

The results obtained with various receivers have a range of 2 to 3 per cent.; but for any one receiver the variations in the measurements are considerably less than 1 per cent. It was found that the presence of atmospheric humidity had a marked effect upon the value of the radiation constant. The general performance of the radiometer is sufficiently satisfactory to warrant the attempt to operate the whole radiometric outfit in an evacuated enclosure, thus eliminating absorption by carbon dioxide and water vapor. Another difficulty (in addition to atmospheric absorption) is the temperature scale. Much time has been spent in attempting to obtain an accurate agreement in the measurements of the temperature of a black body as determined by means of thermocouples and an optical pyrometer, both of which were calibrated at the melting points of metals. In view of this difficulty, the coefficient of total radiation is to be determined by measuring the radiation from suitably constructed black bodies heated to the melting points of metals. At the rear of the radiator is a thermo-

[1] Abstract of a paper presented at the Columbus meeting of the Physical Society, December 27, 1915.

pile (with a circular receiver) which is used in maintaining a constant temperature radiometrically instead of thermometrically, with thermocouples, as has, heretofore been the custom. Equipped with this outfit, as soon as a vacuum spectrometer can be provided the facilities will be at hand to redetermine the constant (C_2) of spectral radiation.

The radiometer is a *null* instrument which evaluates radiation energy in absolute measure. This enables one to determine various subsidiary physical constants. Auxiliary apparatus has therefore been constructed for operating the device as a physical photometer by means of which it will be possible to determine the luminosity, and the radiant luminous efficiency of a black body at various temperatures; also the mechanical equivalent of light—all of which constants have been discussed recently by Pirani and others on the basis of the pioneering work primarily of Lummer and Pringsheim done some twenty years ago.

All the subsidiary measurements are in terms of the radiation of a black body at a known temperature so that the exact value of the coefficient, σ, is of secondary importance. The forthcoming paper contains also a summary of the most reliable determinations of this constant as obtained by various observers. It is shown that all the measurements give a value of the constant of total radiation which lies close to $\sigma = 5.7 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$

BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE CONSTANTS OF RADIATION OF A BLACK BODY.[1]

BY W. W. COBLENTZ.

THE present communication gives the results of a recomputation of the constants of spectral radiation of a black body, which data were presented at previous meetings of this Society. This recomputation was necessitated as the result of the adoption of a new and apparently more reliable calibration curve of the fluorite prism used in the work, and as a result of the discovery of a small error which was found in the previous computations. The results of the present computations give a value of $C = 14{,}369$ and $A = 2{,}894$.

The data of other investigations are summarized and it is found that they lie close to $C = 14{,}350$.

At the Columbus meeting the value of the coefficient of total radiation was reported to be $\sigma = 5.72 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$ Hence, if theoretical considerations are to be relied upon, this value of the coefficient of total radiation indicates that the coefficient of spectral radiation is of the order of $C = 14{,}322$ as compared with the experimentally determined value of $C = 14{,}369$. This is a difference of only 0.3 per cent. The small outstanding discrepancy is probably due to uncertainties in the calibration curve, and in the determination of the value of σ.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 21, 1916.

From a consideration of the data now available it appears that the values of the constants of spectral radiation are close to

$$C = 14{,}350 \text{ micron deg.},$$
$$A = 2{,}890 \text{ micron deg.},$$

and that the coefficient of total radiation is of the order of $\sigma = 5.7 \times 10^{-12}$ watt cm.$^{-2}$ deg.$^{-4}$ This indicates that the constant h, of the quantum theory is of the order $h = 6.56$ to 6.57×10^{-27} erg sec., which is Millikan's latest value.

WASHINGTON, D. C.
March 24, 1918.

ERRATA AND ADDENDA.

Vol. V., March, 1915, p. 238, article by D. L. Webster, entitled "The Intensities of X-Ray Spectra"; the author adds the following correction:

In the REVIEW for March, 1915, I published an article showing that if the length of a wave train of X-rays is small compared to the distance in which a crystal is uniform enough to give good interference, one might expect the intensities at corresponding points in X-ray spectra to diminish with increasing order somewhat faster than the inverse square. More recently, however, a more detailed examination of the question, by Fourier's series as well as by the method of that paper, has shown that under these circumstances the widths of the reflections should appear proportional to the order. Since, instead of that, the widths appear constant, this hypothesis cannot be correct, and the wave trains must be distinctly longer than the uniform stretches of crystal structure. Undoubtedly the correct explanation of the intensities must be that of Bragg's Bakerian Lecture of 1915, based on the distribution of reflecting electrons in the atom.

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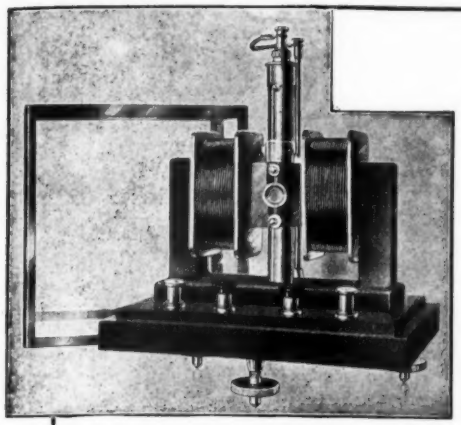

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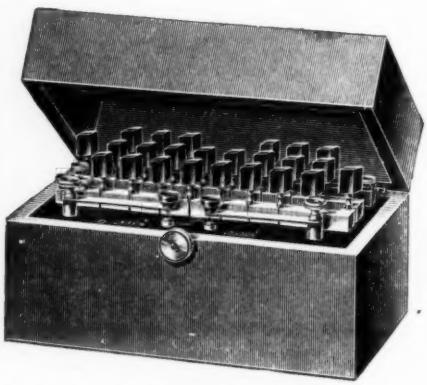

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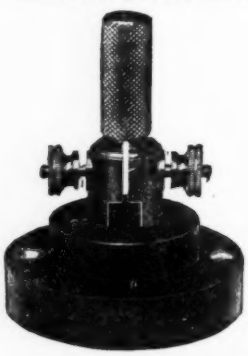

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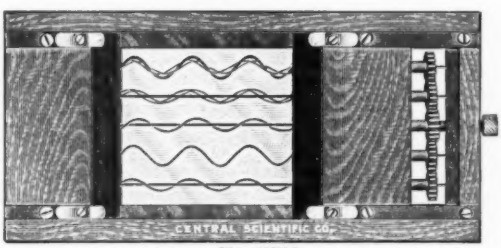

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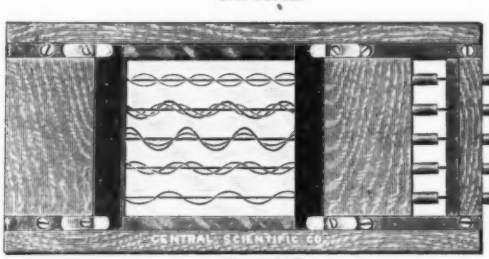

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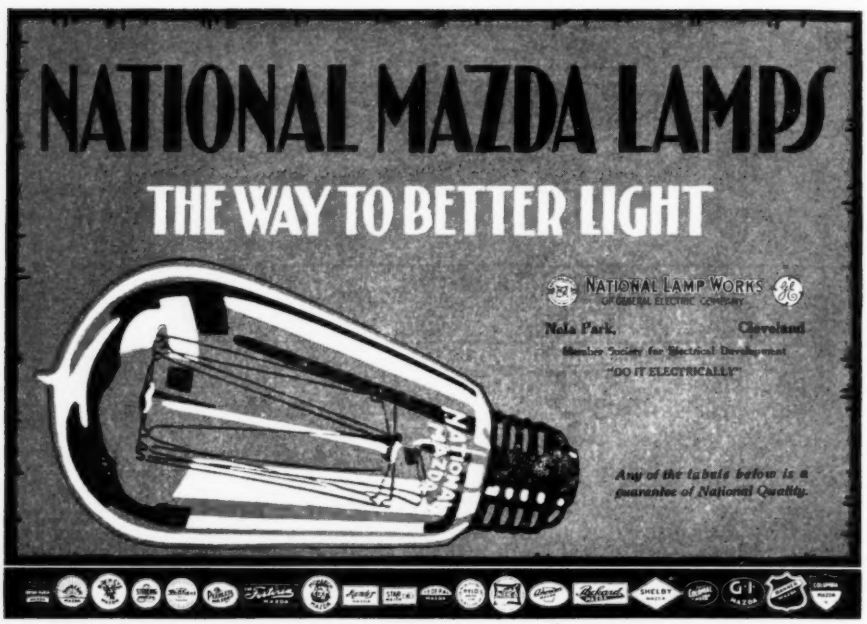
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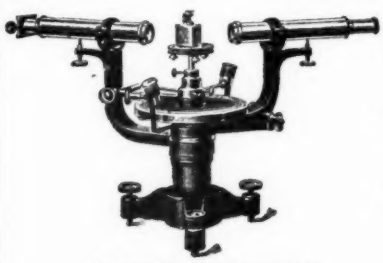

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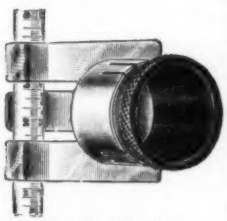

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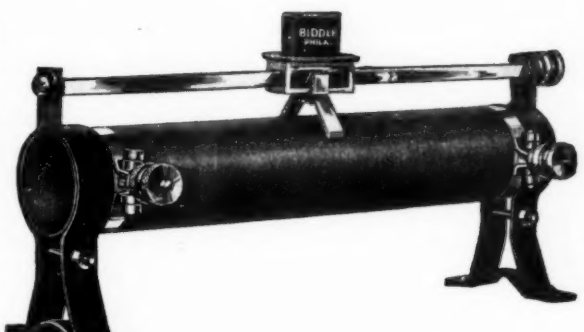

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